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A study of the KappAbel mathematics competition

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Research report

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- Research report

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Preface

The research project reported in this book is initiated by the Nordic Contact Committee (NCC) of the 10th International Congress on Mathematics Education (ICME-10). In the spring of 2004, they called for research projects on the Nordic mathematics competition KappAbel. The projects were to address significant questions about results of the competitions, in particular “possible changes in student attitudes towards mathematics, as well as possible changes in the practice of involved teachers”.

The empirical focus of our study was the KappAbel mathematics competition in Norway in the academic year 2004-2005. We followed the competition from September 2004, when the theme “Mathematics and the Human Body” was announced, over the first and second round in November and January 2005, until the national final in April 2005. We did the first survey in December 2004 and the last observations and interviews in November 2005.

The research project “Changing views and practices” is partly financed by a grant from NCC, and partly by the Norwegian Center for Mathematics Education where the project is situated. The members of the research team are Tine Wedege (research leader), the Norwegian Center for Mathematics Education and Malmö University, Jeppe Skott, the Danish University of Education, Inge Henningsen, the University of Copenhagen, and Kjersti Wæge, the Norwegian Centre for Mathematics Education (research assistant). Between us we have nine months of work financed within the project budget.

We want to thank, first of all, the teachers and the students who gave their voices to this study and who spent their valuable time participating in surveys and interviews. Besides we thank Roald Buvig, Frolands Verk, project leader of KappAbel, and Knut H. Hassel Nielsen, NTNU, responsible for the web in the competition, for statistical information and contact to the teachers in KappAbel. And thank you to Svein Torkildsen, Samfundets Skole, Kristiansand, who constructively commented on the questionnaire and gave an interview, and to Anna Kristjánsdóttir, Agder University College, who also gave an interview.

Many thanks to Tor Nielsen, who illustrated KappAbel with interest and nerve, and to Inger Sandnes, Trude Holum, Bjørg Riibe Ramskjell, Astrid Liestøl Henningsen, Anja Angelsen and Anna Folke Larsen who did transcriptions, translations and proofreading.

Finally we want to thank Ingvill M. Stedøy and her colleagues at the Norwegian Center for Mathematics Education who always showed vital interest and support, not the least Merete Lysberg, who organised the sending out of the first survey, effectively supported by Margit K. Jensen and Ingunn Seem, who also coded the questionnaires; and Randi Håpnes, who has kept a check on the expenses.

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Content

Chapter 1

The KappAbel Study

The study addresses the issue of potential and perceived influence of the KappAbel competition on the mathematical attitudes and practices of the participating teachers and students. In chapter 1, we will present the KappAbel competition: its organisation in three phases (two qualifying rounds; project work and semi-finals; final) and some examples of mathematical problems and project work. We will present the visions of KappAbel as stated at the official web-site and by three mathematics teachers and researchers who are deeply involved in the competition. Furthermore we will present and discuss the problem complex around the KappAbel competition, our research interest and our research questions. But firstly we will give a short presentation of mathematical instruction in the Norwegian primary and lower secondary school.

1.1 Mathematical instruction in the Norwegian school

A reform of the Norwegian ten-year basic school was initiated in 1997, also introducing new syllabi for all school subjects. According to the mathematics syllabus, instruction is to create interest and insight in the subject on the part of the students, so as to develop their proficiency in using it in daily life and in other school subjects (L97,1996). The teachers are to introduce practical connections, examples and working methods in order to provide their students with opportunities to develop positive attitudes towards the subject, and instruction is to assist the students in developing their proficiency in using their mathematical qualifications for communicative purposes in a modern society. Also, the students are to experience mathematical learning as an investigative and reflective process.

These intentions are outlined in slightly more detail for different age groups in a section of the syllabus describing the types of classroom activities envisaged for school mathematics. In the first phase of primary school (students from 6 to 9 years of age), play is a key term, and instruction is to take its point of departure in the students’ everyday experiences. The syllabus also encourages cross-curricular activities, integrating mathematics with other subjects. In middle school (students from 10-12), mathematics education is to be linked to or embedded in practical activities. Theory and practice are to be linked by using play and games, and by working with nature and the students’ local environment. From such starting points, each student is to be challenged in a variety of ways and at her own level of expertise, while engaging with the more abstract aspects of mathematics. At the lower secondary level (students from 13-15), there is greater emphasis on the more formal and abstract sides of mathematics and on the applications of mathematics in society. Also at this level, instruction is to take its point of departure in the students’ own experiences. At all levels, the students’ involvement in genuine mathematical activities is emphasised. (L97, 1996).

From 1998-2003 the Norwegian Research Council conducted an evaluation of the 1997-reform on behalf of the Ministry of Education and Research. Alseth et al. (2003) did the study on mathematics in primary and lower secondary school. In their interpretation, there are five main expectations concerning mathematics instruction in L97. These are that the school subject is to

1) be practical;
2) emphasise conceptual development;
3) be investigative;
4) be communicative and cooperative;
5) adopt also historical and cultural perspectives on the subject.

Although these five points have been the basis for supervision and in-service education of teachers, and although teachers are generally pleased with the syllabus, the intentions are not to any great extent reflected in instructional practice. Based on interviews with teachers and on classroom observations, Alseth et al. conclude that mathematics instruction follows a traditional pattern consisting of teacher presentation of problems from the students’ homework and of new content to be covered followed by the students’ individual work on textbook tasks. This means that none of the five expectations mentioned above are met to any great extent. For instance, links between mathematics and non-mathematical contexts are peripheral to the bulk of what happens in school mathematics, and whenever there are such links, they are mainly in the form of textbook presenting one isolated situation after the other in independent tasks. More specifically, mathematics is rarely used to work on themes or projects from outside the subject itself. Also, there is little cooperation and discussion, as the textbook tasks leaves little room for communication except from stating whether a result is right or wrong. The school subject is isolated and textbook centred, and both teachers and textbooks emphasise mastery of specific skills rather than understanding of mathematics as a coherent structure.

Alseth et al. also mention that students lose interest in mathematics as they move up through the grades. The students see little need for the specific knowledge and skills emphasised in mathematics instruction. Also, instruction does not to any great extent take the students' different abilities into account, and the implicit expectation seems to be that all students are to learn the same mathematics in the same amount of time.

According to Alseth et al., there are also examples of classroom instruction more in line with L97. These alternative practices vary considerably in quality, but they share an emphasis on investigations, communication, cooperation, multiple and varied teaching-learning aids and connections to the students' everyday life as expected in L97. They are, however in stark contrast to the traditional pattern outlined above.

1.2 Introduction to KappAbel

KappAbel is a Nordic mathematics competition for students in lower secondary school. The overall aims of the competition are (1) to influence the students' beliefs and attitudes towards mathematics and (2) to influence the development of school mathematics. The aim of the present study is to contribute to an understanding of the extent to which KappAbel meets these aims.

The name "KappAbel" is first and foremost about being capable (Norwegian: kapabel). The name also illustrates that this is a competition (Norwegian: kappestrid). Last – but not least – the name is meant to honour the Norwegian mathematician Niels Henrik Abel (1802-1829). As a matter of fact the first KappAbel competition was held in 1997 in Froland Kommune, the municipality where Abel was buried. As an initiative in the World Mathematical Year 2000, KappAbel grew into a national competition and around 2000 students participated this year in the first rounds. This happened in cooperation with the Norwegian organisation of mathematics teachers, LAMIS (Landslaget for Matematikk I Skolen), and Department of Mathematical Sciences, the Norwegian

University of Science and Technology. Already in 2001, 10.000 students participated in the first two rounds.

From 2002, KappAbel has developed from a national to an international competition involving all the Nordic countries. The other countries were invited to participate via the Nordic Contact Committee of 10th International Congress on Mathematics Education (ICME-10), and the second Nordic final took place at the ICME-10 conference in Copenhagen in 2004.

Three basic ideas

When developing from a local into a national competition, the concept of KappAbel changed. The competition is now based on collaborative work in whole classes, and the class counts as *one* participant. The problems to be solved are non-routine investigative tasks, and the classes that progress to the semi-final have to do a project on a given theme, e.g. “Mathematics and music”. At the semi-final each class is represented by a group of four students, two boys and two girls.

It follows that KappAbel is based on the three following ideas:

1. The whole class collaborates and hands in a joint solution.
2. The class is doing a project work with a given theme.
3. There are two boys and two girls at the final team.

KappAbel, then, focuses on investigations and project work and signals that mathematics does not consist merely of closed lists of concepts and procedures with which to address routine tasks. Also, the emphasis on collaboration in whole classes suggests that there is more to mathematical activity than individuals engaging the development or use of such concepts and procedures. In this, KappAbel seems to be in line with international reform efforts.

1.3 Practice of KappAbel

KappAbel is a competition in Mathematics for school classes in the Nordic countries. In Norway and Iceland the participating classes are grade 9; in Denmark, Finland and Sweden they are grade 8. The students of the participating classes in the school year 2004/05 were born 1991. There are two qualifying rounds in the competition, taking place in November and January. These are web-based and consist of joint problem solving activity in the students' normal classrooms. The problems are to be solved by the whole class within 100 minutes at some time during the two weeks in which the website is open. The teacher has to download the problems from the internet (using a user's name and a password given him/her from KappAbel), and the answers must be registered the same way.

On the basis of these two qualifying rounds a number of classes qualify for the national semi-final. In Norway one class from each fylke (county) qualifies. These 19 classes have to prepare a project on a theme given by KappAbel. In 2004/05 the theme was "Mathematics and the Human Body". The students present the results of their project work in a report, a log book and at an exhibition, when they meet for the semi-final. A team consisting of 2 boys and 2 girls represents each class at the semi-final. Apart from presenting their project, these students engage in another problem solving session. Based on an evaluation of the project, the project presentation and the problem solving activity at the semi-final, the three best classes are selected to meet for the national final. At the national final, these three classes are represented by the same four students as in the semi-final, and they are to do another round of non-routine, investigative tasks. (Sources: www.KappAbel.com and Holden, 2003)

The role of the teacher

According to the KappAbel website, the mathematics teacher has the role as a secretary. He/she has to:

- Register the class for the competition
- Download the problems for the two qualifying rounds
- Submit the answers for her/his class electronically via the KappAbel website and as a printed copy to the organisers of KappAbel in Norway
- Give the preliminary results to the students
- Give the students the official results for each qualifying round after they have been presented on the website
- Help the students organize the project work and be their coach and advisor while they are working with the project, if the class qualifies for the semi-final.

It is emphasised that the project should be done entirely by the students.

Phase 1 – round 1 and 2

The competition starts with two web-based qualifying rounds taking place in November-December and January respectively. The mathematics teacher downloads the problems from the web-page. (See examples of problems in figure 1.1 and 1.2.) The students are allowed to use 100 minutes, including reading the problems, and decide how the class will respond answer to each problem. All kinds of materials, tools and aids can be used when solving the problems, but communication out of the classroom (internet, cell phones) is not allowed during the two qualifying rounds.

The teacher submits the solutions of the class immediately after they have finished. In the qualifying rounds the solution to each problem is awarded between 0 and 5 points. A wrong solution gives 0 points, while no solution gives 1 point. A solution may be given between 1 and 5 points if it is partly correct.

Figure 1.1 Problem no. 3 from Round 1 - KappAbel 2004/05

3. SUMS AND PRODUCTS

The sum is 12. How big can the product possibly be?

There are many ways in which to write 12 as a sum of positive integers.
8 + 4 and 3 + 2 + 7 are just two of many.

If we multiply the summands of these two sums we get
8 * 4 = 32
3 * 2 * 7 = 42

Find the sum which gives the biggest possible product.

Write down your answer.

No. 1	No. 2	No. 3	No. 4	No. 5	No. 6

Sum	Product
12	

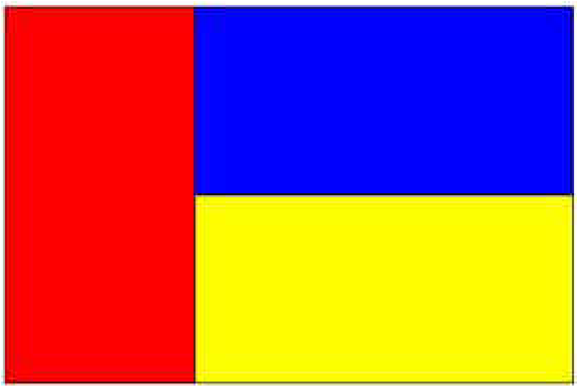
(You may choose as many as 6 integers to represent 12 as a sum)

If you find the biggest you get 5 points. The second biggest gives you 3 points, and the third biggest gives you 2 points. Any other answer gives no score.

At the KappAbel website, the teacher might find some pieces of good advice on how to organise the qualifying rounds. To have as many students as possible taking an active part in the competition, it is recommended that the class is organized in groups of four while working with the problems. Each group starts with a different problem and work their way through as many as possible. Possible disagreements are to be discussed by the class before submitting their solutions. It is suggested that the teacher draws a diagram on the blackboard or on an overhead transparency with the numbers of the problems, and the alternative solutions in order to facilitate discussion and reach an agreement.

Figure 1.2 Problem no. 2 from Round 2 KappAbel 2004/05

2. FROM RECTANGLE TO SQUARE



Three rectangles of the same size and shape have been put together to form a greater rectangle. The area of the great rectangle is 168 cm^2.

Find the area of a square that has the same perimeter as the great rectangle.

Answer: ☐ cm^2

Phase 2 – project work and national finals

Each Nordic country arranges semi-finals and national finals. The semi-finals and finals take place on the same days, with the same problems and the same organization in all the participating countries. When a class knows that they are proceeding to the semi-final, they pick four students, two girls and two boys, to represent the class for the project presentation and the problem session of the semi-final. Three of these teams also represent their class in the national final. In 2005, the national semi-finals and finals took place in April.

The project work

An important part of the semi-final is a project with a given theme, which is published at the beginning of the academic year. Other classes are encouraged also to do the project. Earlier themes have been "Mathematics in local traditions: culture, arts and crafts", "Mathematics in nature", "Mathematics in games and play", "Mathematics in sports" and "Mathematics and technology". The theme in 2003/04 was "Mathematics and music", and in 2004/05 it was “Mathematics and the Human Body”.

According to the KappAbel website:

- A project is best if the students start from something they are curious to investigate. During the work to find an answer to the problems, something unexpected always emerge and the questions have to be changed or new questions have to be formulated.
- The project themes should allow for and encourage students’ imagination and creativity. However, mathematics has to be in focus and made visible all through the project work. Whatever the students choose to investigate, they have to adopt a mathematical perspective.

For the classes in the semi-final, the project work is assessed by a jury of mathematics teachers, and it counts 50% of the total score. The basis for the assessment is provided by:

- A process log about the project work
- A mathematical report
- A public presentation of the project with an exhibition and an oral presentation.

According to the KappAbel webpage the *process log* is addressing the following topics:

- How did the class arrive at the problem formulation that formed the basis of its work?

- What work was done in each lesson or unit of work in the entire project period? What mathematical problems arose? And what was done to solve these problems?
- How well did the group/class co-operate?
- What is the class' evaluation of the efforts they have put into the project?

The length of the process log should not exceed three typewritten A4 pages.

- *Exhibitions: Mathematics and the Human Body*

The mathematical *project report* must include

1. A front page listing the topic, the class, and the date of completion.
2. A preface clarifying the problem complex and explaining why it was chosen.
3. A main part describing the work of the class on the problem. This part should describe the questions that have been addressed, the lines of reasoning that have been followed, and the calculations that have been conducted, as well as the results that follow from these calculations and lines of reasoning. It is an added advantage, if the class manages to

demonstrate how new discoveries and problems turned up during the course of the project.

4. A conclusion: A short summary answering the questions and addressing the problems that were raised as the basis for the work.
5. A list of references: The title and author of all materials used during your work on the project: books, films, web sites, leaflets, interviews, etc.

The length of the project report may be up to six typewritten A4 pages, excluding the front page and the list of references.

The *public presentation* of the project work includes:

A. Exhibition. The exhibits must fit into a box no larger than a 1/2 cubic metre. The exhibition space allotted is a table surface measuring 88,0 cm x 75,0 cm. In addition, a wall space measuring 104,0 cm x 88,0 cm (height x breadth) will be available. (See photos p. 15.)

B. Oral presentation. The four students representing the class in the semi-final will give a 10-minute oral presentation of the project to the assessment committee and the other participants in the semi-final. The oral presentation takes place after the problem solving part of the semi-final.

The *assessment* of the process log and project report emphasises the following:

- originality, creativity and deep understanding;
- the ability to communicate how the work on the project has developed;
- the extent to which the written products show that you have detected and understood the mathematical component of the topic.

The assessment of the exhibition and presentation emphasises:

- the extent to which the mathematical components are evident in the public presentation;
- the extent to which the presentation will stimulate interest in mathematics among other children and youngsters through the artistic design of the project exhibition. (See the KappAbel website: www.kappabel.com)

Organisation of semi-finals

The project is evaluated by a project jury and it counts 50% of the semi-final. As mentioned above creativity, originality, cross subject solutions and mathematical content is valued. The other 50% of the semi-final is a problem session. The four representatives of each class jointly solve six to eight problems in 90 minutes. No aids are allowed in the problem session of the semi-final. The score in the semi-final is independent of the results of the qualifying rounds.

In Norway the project presentation and exhibition of the projects took place in the library of the town of Arendal in April 2005. The problem session of the semi-final was organised in a big hall with a table for each of the 19 teams, placed in two rows. Between these rows is another row of tables where envelopes with the problems and the necessary equipment are placed. A young representative of KappAbel at each of the middle tables is to assist the students and to make sure that everything is OK throughout the competition. When the competition begins, one student from each team rushes up to the table to get the first envelope. The students solve the problem together, write down their solution on the answer sheet, and put it back in the envelope. The envelope is handed in before they pick up the second envelope; etc. The teams decide for themselves how much time they will spend on each problem. They have 90 minutes to solve all the problems.

A KappAbel representative responsible for the problems has to be in the hall while the students are working. He/she explains the rules for the teams and answers questions from the students throughout the competition. When the 90 minutes have passed, the answers are given to a jury. They evaluate them before the project presentation. The scores from the problem solving session will not be given to the students until after the project presentation.

- *National final at Arendal*

The national final

The day after the semi-final, the four students from each of the three best classes meet for the national final in a music hall, a theatre or something similar. The score in the final is independent of previous results. The teams solve six demanding KappAbel problems and no aids are allowed during this session. While the teams are struggling on the stage, the audience (the other teams from the semi-final and a number of local 9 grade classes) is invited to try to solve the problems. The final is not over until it is clear who is number one, two and three, i.e. the teams will continue solving problems. There are two judges – one experienced mathematics teacher and one professor in mathematics from the Norwegian University of Technology and Science.

Phase 3 – Nordic final

The winning team from each country participates in the Nordic final in July. The team presents their class project from the semi-finals and compete in a problem solving session.

During the Nordic final the project presentation is in English. Each team has 20 minutes to present their project orally. The problem session of the final is of the same kind as the national finals, but the problems are presented in English. The teams will be given a written version of the problems in their own languages. After the answers are handed in to the jury, the teams will present their solutions orally in English.

1. 4 Visions of change

KappAbel differs from other mathematics competitions in many respects. Already the three basic ideas of whole class collaboration, project work and teams of two boys and two girls indicate the differences. The official KappAbel homepage presents some visions of KappAbel that go beyond the competitive element itself:

- “With KappAbel we wish to inspire students in a way that they do not loose interest of mathematics in a problematic age at school, but that they on the contrary keep and develop their relationship with mathematics.

- With KappAbel we wish to show that there is more to mathematics than the anwer, the one right answer. Mathematics also involves discovery, fantasy, wonder and collaboration.
- With KappAbel we wish to contribute to developing a better and more adapted pedagogy in mathematics through discovery, fantasy, wonder and collaboration. Especially we think that our project problems might contribute, but also that the competition form it self to a large extent sets the scene for discussion and collaboration in a subject that for so long has been marked by individualism.
- With KappAbel we wish to focus on the great, Norwegian mathematician Niels Henrik Abel (1802-1829). Despite his short life time he is estimated to be one of the world's biggest mathematicians ever, and we hope that our focus on him will be a source of inspiration to the students." (www.kappabel.com)

At the same homepage you find some political statements about decreasing enrolment to science and mathematics education (No: realfag), and the vision is formulated like this:

- "It is our hope that KappAbel – in the spirit of Niels Henrik Abel – will contribute to at better enrolment in science and mathematics education. As it is an incontrovertible fact that Niels Henrik's contribution to mathematical fundamental research has been an important guide in to day's science and technology. You could rightly claim that his mathematical research is one of the conditions for what we know today as an intellectual and a high technology society." (www.kappabel.com)

In the research project we do not focus on the importance of Niels Henrik Abel, who gave name to and is an guiding symbol of the competition, neither on the question about students' interest in doing mathematics in the future. Our focus is the extent to which KappAbel is successful with regard to the two following dimensions of the overall purpose of the competition:

1. To influence students' affective relationship with mathematics (beliefs and attitudes) and the development of pedagogy in school mathematics.
2. To emphasise the collaborative and creative aspects of mathematics in schools.

In order to obtain a clearer picture of the visions of KappAbel we asked Ingvill Stedøy, director of the Norwegian Center for Mathematics Education, and Svein Torkildsen, mathematics teacher at Samfundets Skole in Kristiansand, to elaborate on it. Together they developed the present concept of KappAbel, and they produce all the mathematical problems for the competitions. We also interviewed Anna Kristjánsdóttir, professor at Agder University College. She is responsible for KappAbel in Iceland, the only Nordic country except Norway where the competition is nationwide.[1]

Changing views among the students?

We asked Ingvill Stedøy, Svein Torkildsen, and Anna Kristjánsdóttir to respond to two questions. The first one was: One of the primary objectives of KappAbel is to change students' views and conceptions of what constitutes mathematics and mathematics teaching. What changes do you envisage?

Ingvill Stedøy has a vision of students whose conception of mathematics has become considerably wider:

> I envisage students finding out that they can cope with considerable challenges without the help of the teacher. The fact that they are presented with problems that differ from the ones they are accustomed to, may give them an experience of mathematics as not only learning formulae applicable to specific types of tasks - but that it is also a matter of improving their flexibility in terms of strategies and choices of method.
>
> When it comes to the students who progress to the semi-final and carry out the class project, I think this will give them an experience of project work which they've never had before. Since the focus is on mathematics, they'll be able to see mathematics in new contexts. Hopefully, they'll discover mathematics as a subject relevant to a number of contexts they haven't considered it a part of before.

Svein Torkildsen envisages students who want to be challenged:

> I've experienced that students have a positive conception of this approach to working with mathematics. They sometimes put pressure on the teacher to supply these types of problems. "KappAbel problems" has become a concept among many of them. Together with the activities the students face from Year 1 on the Mathematics Day through the booklet from the National Association for Math Teachers (LAMIS), the KappAbel type of problem challenges the students to think extra hard.
>
> The KappAbel project is probably too much for most teachers. They have to put much of their energy into the project. Not necessarily in terms of time, but in the sense that class work is very intense during the project period. The teacher is supposed to provide inspiration; make sure that the social element is in place and

[1] Ingvill Stedøy and Anna Kristjánsdóttir were also members of the Nordic Contact Committee (ICME-10) who initiated the KappAbel Study (see the preface).

> that the co-operation works as it should; and maintain the confidence among the students that they have a project to be proud of.
>
> We see that the semi-finalists have put a lot of work into their projects. At our school, we have experienced more than once that working on this type of project not only represents a challenge in terms of the subjects involved, but that the process in itself has a very positive influence on class dynamics. But classes may use the project idea as a basis for work without putting too much work into the project itself. It’s a matter of coming up with a problem description, gathering data, experimenting and researching, and trying to arrive at a conclusion. Most of them will perceive this as a small step in the right direction in relation to an open problem. I see this open task – or for that matter a closed task with several solutions – as a link to a small project. Open problems are a good instrument for awakening the students’ inquisitiveness and thus for developing a learning atmosphere characterised by co-operation and dialogue about mathematical problems. The question is whether the national tests will inspire people to work this way…?

Anna Kristjánsdóttir sees KappAbel as a tool for the teachers:

> First, I think it is somehow over-ambitious to imagine that student views would change. If all you do is give the students KappAbel, nothing much happens in terms of learning mathematics. It takes more than that. I think the main thing is that KappAbel provides the teachers with tools so that they start to perceive more clearly both what the students can do, and what their interests are. Which isn’t something they can see in their everyday teaching. Which in turn creates resonance. The teachers have said things like: ”I was so surprised that they were actually capable of reasoning.” Which is something many teachers haven’t seen before, and it’s happening even in classes where all the students have experienced failure in mathematics. One teacher said that her wish had been granted – that she had been given a gift that could stimulate them, and that they actually managed to meet the challenge. This is the possibility I envisage.
>
> Secondly, there is the class project. This too provides the teachers with a tool. Many students don’t even see mathematics in the subjects where it’s frequently applied. This emerges clearly from my research in Iceland and Norway. And many students have a narrow concept of how they apply mathematics in their everyday lives. It is mostly something to do with money. So the class project creates a whole new possibility. If one ensures that the project is wide enough to catch everybody’s interest in one way or another. Through the class project we also supply the teacher with a tool for initiating work that can generate attention to mathematics. And we’ve seen clear signs of how the class project unites the class.

> The three aspects of KappAbel – solving the problems through reasoning, co-operating as a class, and then the class projects – provide something that curriculae, teaching materials or continuing education have been unable to give the teachers to the same extent.

Changing school mathematical practices?

Our second question was formulated thus: Another primary objective of KappAbel is linked to a different school mathematical practice. How do you imagine a different school mathematical practice? What is your vision of a changed school mathematical practice?

Ingvill Stedøy envisages active students who are busy solving problems:

> My vision is that teachers allow the students to be the main protagonists in their own learning processes. By that I mean that they should be active and conscious in relation to learning mathematics. They should be taken seriously, and their initiatives should be welcomed and taken seriously. It is important that the students are given challenges adapted to their own level, that they are given problems and challenges that generate participation, that have the right level of difficulty, and that invite mathematical discussions with their peers. I envisage teachers who have had

> classes participate in KappAbel looking for new ideas for lesson plans from sources other than the set textbook; and as having gained the confidence that the students are motivated by and have a lot to contribute when they're allowed to work on problem-solving tasks, and that they can learn lots of mathematics through working on mathematical and inter-disciplinary projects.

Svein Torkildsen wishes for students to become producers rather than consumers:

> First, it's about getting the students to ask questions of a mathematical nature. For example, how do we construct the golden section? Second, it's about finding out what mathematical tools and principles we need to apply in order to work on the problem. The perspective of application takes centre stage. Here, we bring in the interpretation of our findings: What, for instance, do the statistics and the numerical proportions tell us? In a class where many students are involved in the same project, communication is vitally important. Communication is the third aspect of school mathematical practices where I envisage changes – taking us away from the exercise paradigm. Some call it becoming producers, rather than being consumers of other people's exercises. This is a good project for progress.
>
> This type of mathematical practice exists in today's schools, but I don't think it's very common. But I happen to think that the project is not the aspect with the greatest impact: I have greater faith in the type of problems found in the semi-finals and in the final of KappAbel. Partly also in the qualifying problems. The point is that the problems should provide room for exploring. Many teachers find it interesting to work on such problems. They see, after all, that the students like finding out about things. Here at the semi-finals I heard some students say "That was cool" when they handed in their completed problems. Imagine getting that sort of feeling!

Anna Kristjánsdóttir wants to make mathematics more visible, and to put the emphasis on reasoning in the learning of mathematics:

> My vision about different school mathematical practices is actually pluralistic. In terms of mathematics as such, I want the students to learn to live with it and to enjoy it. That they discover that mathematics isn't something that happens just like that (snap), but that mathematics is something one develops in one's thoughts. That bothering one's brain with a mathematical problem can actually be enjoyable: "No, no, don't tell me the answer! I want to be allowed to think for myself." You need that sort of attitude to mathematics, because these days we have tools to do things fast, given that we have first considered what the problem is all about.
>
> Secondly, I want to counteract the tendency for mathematics to be so hidden away. That school mathematical practices should involve immersion in fields that don't

> seem to have anything to do with mathematics – in order to find out what mathematics has to do with that field. The class project where they first find an exciting area to immerse themselves in, then work on it as a whole during class, in order to finally convey their findings to others, is – after all – an all-inclusive activity. And the students take responsibility both for the process and the results.
>
> Young people should be allowed to build a personal relationship to mathematics – a very important factor in avoiding the anguish experienced by many when they are expected to learn something by heart without understanding where it came from, and how it came about.
>
> The third point I want to mention is that they learn to understand mathematics as a process of reasoning, and that they enjoy reasoning in their learning. Research among university students trying to solve mathematical problems by remembering techniques rather than analysing the problems shows that reasoning as part of learning needs to be made visible, but that it should not be left to the young people alone. I consider KappAbel a good instrument, where the young really wish to use reasoning in the problem solving process. The teacher can then build on this, so that the students also want to use reasoning in their learning beyond KappAbel. By seeing that the students are able to use reasoning in mathematics, the teachers also understand that the students can apply reasoning in a wider context.

It is not difficult to find common themes in these three sets of visions for school mathematics. There is in all of them a definite attempt to expand the notion of what the subject of mathematics is by including and emphasising process aspects of the subject. This process orientation includes moving beyond practising procedures for solving particular types of problems. Also, it inserts elements of autonomy, reflection and flexibility in the ways students manoeuvre in relation to mathematics. Wording and investigating problems in a variety of situations, much beyond the ones that dominate school mathematic today, is a key concern. Not surprisingly, these visions for the subject are to be found also in the ones for the teaching-learning processes that are to lead to their realisation. The process orientation and the literally creative aspects of the subject are to be inherent elements also of the practices of mathematics teaching and learning.

The three visions are also similar in what they do not mention. There is little indication that there is a need to change some of the traditional contents of the subject. Also, there is no mention of if and how the envisaged changes relate to and should or should not be balanced by some of the more traditional aspirations and teaching-learning processes of school mathematics (cf. e.g. Sfard, 2003). We shall not discuss these issues in this report. Instead we shall take the visions as espoused above at face value and investigate if and how KappAbel may contribute to their realisation.

It is the ambition, then, that KappAbel may facilitate changes in school mathematics practices along these lines. These are high hopes. The ambition of the present study is to investigate if they are too high.

1.5 Problem field, research interest and research question

In this section, closing chapter 1, we present some central problems and distinctions from the international research on mathematics competitions. We look at the KappAbel competition in the context of the international reform efforts in mathematics education and on its role as an external source of influence on teaching and learning processes in mathematics classrooms. Also, we introduce belief research in mathematics education, and finally we give a first presentation of the research questions of this study.

International research on mathematics competitions

"All of the above activities [mathematical competitions of various kinds] have a positive effect, direct or indirect, on the teaching and learning of mathematics and in attracting students to the study of mathematics." This statement stems from the policy paper of the World Federation of National Mathematics Competitions (WFNMC, 2002). In Discussion Group 16 (The role of mathematical competitions in mathematics education) at ICME-10, the Organising Team had decided to focus on two questions: "(1) Do mathematics competitions contribute to widening the gap between mathematics for all and mathematics for the elite, or can the opposite be the case? (2) How can competitions motivate and foster mathematical creativity with students at large?" As described in the preface, the Nordic Contact Committee of ICME-10 had formulated a call for research projects on the mathematics competition KappAbel that should address significant questions about results of the competitions, in particular the question of "possible changes in student attitudes towards mathematics, as well as possible changes in the practice of involved teachers".

WFNMC describes the overall positive effect of mathematical competitions. DG16 focused on two possible results of competitions and the same did NCC. In relation to mathematics education, the crucial issue seems to be something like this: Do competitions contribute to the change of attitudes and practices – or are they just an isolated period of concentration, joy, anxiety, challenge and fun with no consequence for mathematics education in general? It is obvious that these questions cannot be answered in general; any answer needs to take into account the specific design of the competition and the particular context of mathematics education .

ICMI study 16 (2005), Challenging Mathematics in and beyond the Classroom, will present studies of mathematics competitions among other activities and initiatives such as clubs, exhibits, games and projects. In the discussion document for the study, a preliminary answer is given to the question: What is a *mathematical challenge?* The answer given is that a challenge occurs when people are faced with a problem whose resolution is not apparent and for which there seems to be no standard method of solution. Presented with a challenge, then, people are required to engage in some kind of reflection and analysis of the situation, possibly putting together diverse factors. Those meeting challenges have to take the initiative and respond to unforeseen eventualities with flexibility and imagination. Thus the word “challenge” denotes a relationship between a question or situation and an individual or a group. Finding the dimensions of a rectangle of given perimeter with greatest area is not a challenge for one familiar with the algorithms of calculus, or with certain inequalities. But it is a challenge for a student who has meet such a situation for the first time.

KappAbel as inclusive or exclusive competition?

WFNMC (2002) makes a distinction between the following two categories of mathematical competitions: (a) *Inclusive competitions* are of a popular nature, designed for all students to participate, and certainly accessible to average or below average performing students. Such competitions give each student the opportunity to solve simple though often intriguing problems, which are posed in familiar circumstances. These competitions will not usually be set according to a published syllabus. Examples are multiple choice competitions and first rounds of National Olympiads as they are held in some countries. (b) *Exclusive Competitions* are aimed at the talented student. Once again the syllabus of the competition is rarely formal. But the subject of mathematics being so broad, there is vast material of a challenging nature which enables students to deepen their knowledge and command of mathematics without the need to accelerate their study. Examples are national and international olympiads in mathematics.

In Norway, two mathematics competitions are named after Niels Henrik Abel: the Abel contest and KappAbel. *The Abel contest*, which started in 1980, is a contest in mathematical problem solving for high school students. It consists of two rounds and a final. The winners in the final automatically qualify for the International Mathematical Olympiad. The first round of the Abel contest is an inclusive competition while the second round and the final are exclusive competitions. It is obvious that KappAbel intends to be an inclusive competition, and it is given as an example of one in the ICMI 16 discussion document.

In the European context, KappAbel has been mentioned as an example of a competition that can increase all students' interest in mathematics:

> This project is unusual in that the competition is not only national policy but is designed to involve all students rather than those who are regarded as mathematically able or gifted. Rather than students taking part as individuals, moreover, in the initial stage classes participate as a team. Trough collaborative processes opportunities for mediated learning are provide ad the less able gain support from more able students. The early stages make use of computer based activities with whole class collaboration to produce answers which are fed back into the computer for assessment. In this way another of the main disadvantages of competition, anxiety generated by racing against a visible opponent, is avoided.
>
> Other important aspects involved in raising interest and achievement in mathematics are the se of context and the development of higher order thinking skills. The importance of a focus on contextualized practical work in learning and generating interest is addressed in the Norwegian initiative through the use of project work and problem solving in a later round. Thus the important development of higher order thinking skills in mathematics is also taken into account along with the notion of constructivist principles of active learning.
>
> Gender issues are directly addressed at the later stages of the competition as two girls and two boys are selected to represent the class.
>
> A perceived increase in enthusiasm and motivation has been noted through the engagement. (John Dakers, UK, quoted after Holden, 2003:7-8)

The two terms exclusion and inclusion are connected more with mathematics than with any other school subject. The door named mathematics to education and jobs is experienced as closed by many people. A summary of many adults' relationships with mathematics has been formulated in a single sentence:

“Mathematics – that’s what I can’t do.” (Wedege, 2002). From a broad perspective, people’s attitudes and self-perception in relation to mathematics may be socially generated through their lived experiences. However, there is much evidence to suggest that this belief in adults is primarily a result of teaching and learning mathematics in primary and secondary school. The apparent contradiction between many adults’ barrier in relation to mathematics in formal settings and their competences in everyday life is a puzzling issue. KappAbel is not a competition in the traditional sense, as it is based on collaborative project work in rich task-contexts in school classes, and it might potentially create a situation context for changing young people’s attitudes to mathematics and opening the door to mathematics (Wedege, 1999). This was one of the reasons why we engaged in this research project.

KappAbel is in line with international reform efforts

The current reform movement in mathematics education involves “a set of changes in both the conception of mathematics, the understanding of the teaching and learning of mathematics and the priorities of school mathematics in relation to more general aims of education.” (Skott, 2000:17).[2] With regard to mathematics and its learning Skott summarises the following points:

- mathematics is a human construction developed in mathematical communities;
- mathematical knowledge is preliminary and fallible;
- mathematical learning is an activity that requires goal directed mental and possibly physical action on the part of the learning;
- mathematical learning requires the use of experientially and mathematically rich contexts for the students’ activity;
- mathematical learning requires involvement in communicative and other social interactions;

[2] In chapter 2, The Reform Movement in Mathematics Education, in his dissertation, Skott understands the contents of the reform as “the emerging outcome of the reflexive relationship between theoretical and practical contributions”, and he sums up the current reform by presenting and discussing developments in the theory of mathematics education: Lakatos, Davis and Hersh, Ernest and Skovsmose (conception of mathematics) and Piaget, Vygotsky, Ernest, Bauersfeld, Cobb etc. (theories of learning mathematics)

- mathematical learning is contextually framed and influenced by the institutional setting – understood in both a local and broader sense – in which it occurs. (p.39)

The NCTM Standards might be seen as representing the reform movement. In her contribution to “A Research Companion to Principles and Standards for School Mathematics”, Sfard (2003) discusses what she calls “a serious and comprehensive attempt to teach “mathematics with a human face” ” (p. 353) in light of different theories of learning mathematics. As the plurality – *theories* - indicates she takes an eclectic approach in her analysis. She locates 10 needs of the learners which according to these theories are the driving force behind human learning and necessary to fulfil for a successful learning. Among these needs, we find the following five especially suitable to characterise the reform elements in the KappAbel competition:

1. The need for meaning
2. The need for difficulty
3. The need for significance and relevance
4. The need for social interaction
5. The need for belonging

Sfard sees this need for meaning as primary and all others are presented as its derivatives, and she does this with reference to Piaget and Vygotsky among others. The learner today is given the role of an autonomous meaning builder as opposite to the idea of the learner as a tabula rasa passively absorbing externally generated experiences. According to Sfard, the new curricular ideas of the NCTM Standards are imbued with the spirit of meaning making. Of the five major shifts required by the Standards four address the need four meaning:

- toward logic and mathematical evidence – away from the teacher as the sole authority for right answers;
- toward mathematical reasoning – away from mere memorizing procedures;
- toward conjecturing, inventing, and problem solving – away from an emphasis on mechanistic answer-finding;
- toward connecting mathematics, its idea and its applications – away from treating mathematics as a body of isolated concepts and procedures. (NCTM, 1991:3)

KappAbel has the role of an external source of influence on teaching/ learning processes in the mathematics classrooms

Curriculum reform in mathematics – as well as in other subjects - is often considered a project of development and implementation. Development is done independently of the majority of classrooms for which the reform is intentioned. This leaves the implementation part with a major problem: how to ensure that the intentions of the reform manifest themselves in changes in student learning, also in those classrooms that were never part of the development initiative? Normally this problem is addressed by comprehensive curriculum materials writing and substantial efforts in terms of pre- and in-service teacher education. Materials and teachers are then expected to carry the reform with them into the classroom.

With KappAbel the situation is different. It does not consist for instance of a textbook scheme that may be said to reflect the curricular intentions across the content. Neither does it provide an in-service education programme that is to assist teachers in dealing with new topics or with traditional topics in novel ways. Rather, it attempts to insert practices, which are decidedly different from those that dominate mathematics teaching and learning at present, into mathematics classrooms. KappAbel is, then, intended to be a source of influence on mathematics classrooms that may be deemed compatible with other initiatives to reform teaching-learning practices, but that is nonetheless external and alien to the present state of affairs.

Belief research in mathematics education (introduction to the problem)

Belief research has become a significant field of study in mathematics education over the last few decades. As far as teachers are concerned the main emphases have been the relative stability or potential for change of beliefs and the relationship between teachers' beliefs and the classroom practices. These issues are obviously also significant in the present study. However, a number of conceptual and methodological problems have turned up in belief research. We shall later give an outline of how we conceive these problems and how we have dealt with them in the present context (see chapter 2 and 3).

Research questions

The main objective of the research project *Change of views and practices? A study of the KappAbel mathematics competition* is to investigate potential results of the competitions measured in terms of possible changes in participating students' relationships with mathematics and in the classroom practices. I.e. the intention of the study is to develop an understanding of if and how the KappAbel mathematics competition has an influence on the beliefs and attitudes and the teaching/learning practices of the participating teachers and students. This is a question that is closely related to the more general one of the role of external sources of influence on teaching/learning processes: What may initiate and sustain change in mathematics classroom practices and in students' and teachers' beliefs, and what – if any – is the relationship between the two sets of changes? As the point of departure in this study, we claim that people's relationship with mathematics is developed in different communities of practice (family, school, working life, etc.), however the practice of the mathematics classroom is the most significant of these communities (cf. Wedege, 1999). The question we address in the study, then, is whether participation in the KappAbel competition has the potential to influence beliefs and attitudes by influencing the modes of participation in the practices of mathematics classrooms.

This question is discussed and reformulated within the theoretical framework for the KappAbel study in the following chapter.

Chapter 2

Change in ”views and practices”

This study had the subtitle of *change in attitudes and practices*, where students' and teachers' attitudes towards mathematics were broadly understood as their affective relationships with mathematics. During the study we realised that it was necessary to clarify this relationship, and in the process we decided to change the terminology from attitudes to *views*. In the following we shall outline our understanding of *views* and of *practices*. Apart from the obvious aim of presenting our understanding of two key terms in the study, the intention behind the terminological clarification is twofold. First, it may serve as a basis of an outline of our theoretical framework in relation to concepts that are highly relevant to the study. Drawing on a number of scholars in- and outside mathematics education, we intend to clarify the two terms in ways that suggest how we see the study in relation to mathematics education research in general. In order to understand what "change in practices" means, we introduce *didactical contract* as a metaphor. Hopefully, then, the connotations of the terminological clarification indicate the tradition in which we wish to position ourselves. Second, and following from the first point, the terminological clarification presents a rationale for the study and thereby describes how we see the main task ahead of us. In particular it has implications for the design of the empirical parts of the study (see chapter 3). It follows that the terminological clarification intends to explicate what types of issues we are discussing in this report and what questions we are explicitly not trying to address.

2. 1 Beliefs, attitudes, emotions

Belief research has developed into a significant field of study in mathematics education over the last 20 years (e.g. Leder, Pehkonen and Törner (eds.), 2001). It grew out of a recognition that the teacher's mathematical qualifications in and by themselves do not suffice as a basis for understanding his or her role for promoting student learning, not least in classrooms meant to be informed by process views of mathematics and by theories of knowing and learning that acknowledge the significance of the students' constructive activity. Later influences to a greater extent view learning as an element of the students' participation in the practices of the mathematics classroom. These influences further fuelled the growth of belief research, as they challenged the conception of doing mathematics and of mathematical learning as primarily individual endeavours. The overly individualistic perspective on students' and teachers' mathematical activity was challenged, then, by views inspired by more social orientations, either in the form of socio-cultural theory or of symbolic interactionism. With the introduction of these perspectives, belief research needed also to address questions of for instance if and how teachers and students shared the emerging social orientations of the field in general.

Research on teachers' beliefs has attempted to improve present understandings of (i) the character of teachers' beliefs, (ii) how these beliefs develop, and (iii) their possible connection to the classroom practices. Mainstream belief research claims that (student) teachers' beliefs are fairly resistant to change. However, it also suggests that they may develop in line with current reform initiatives, if pre- and in-service teacher education programmes model the types of teaching envisaged in reform documents and involve the participants in long-term collaborative efforts to develop the practices of their own classrooms through continued reflection. Also, there seems to be some consensus that teachers' beliefs play a considerable role for the learning opportunities unfolding in the classroom. Making and applauding this last point, Wilson and Cooney (2002) claimed that there has been a tradition of basing "research on teachers' beliefs […] on the assumption that what teachers believe is a significant determiner of what gets taught, how it gets taught, and what gets learned in the classroom." (p. 128)

For the larger part of belief research, then, there seems to be an expectation of a positive correlation between beliefs and practice, with the former determining or significantly influencing the latter.

However, belief research has also been subject to a combination of methodological and substantial criticism in recent years. This criticism has challenged some of the premises of the field. Lester (2002) raised a radical methodological criticism related to the implicit-explicit dimension of the concept of beliefs. Often considered as deeply rooted, individual mental phenomena, beliefs are described as residing at levels of consciousness that are not immediately accessible to observers. The problem, then, is how to get to know this highly personal realm that is not or only rarely made explicit, and which is sometimes even regarded as implicit by definition (e.g. Pehkonen and Törner, 1996). Using an extreme interpretation, this last suggestion is hardly feasible, as it has as a consequence that any statement of the form 'I believe that …' is false by definition, merely because it can be made. Another interpretation, of course, is that beliefs are considered hidden priorities behind such a statement, i.e. that they are the real entities behind or implicit in the statement itself.

Lester's point was that research on belief-practice relationships runs the risk of becoming a self-fulfilling prophecy. It often contains a circular argument of claiming that certain observed mathematical practices are due to beliefs, while at the same time inferring mathematical beliefs from the very same practices. This risk relates to the discussion of the implicit or explicit character of beliefs, as it stems from an agreement in much belief research that espoused versions do not have a privileged position as an entry point to understanding beliefs. Beliefs are often conceived as propensities to engage in certain practices in particular ways under certain conditions (e.g. Cooney, 2001, p. 21). Consequently they are often inferred from observations of the practices in question rather than from what teachers claim in interviews to be their school mathematical priorities. Lester's point, then, relates to a situation in which the call for multiple methods in belief research is confounded with a type of methodological triangulation that assumes identity between objects researched with different methods. We shall return to this issue in chapter 3 on the design of the study.

Lester made the above point in relation to students of mathematics. If reinterpreted so as to relate to teachers, it is in line with Skott's criticism of mainstream belief research (e.g. Skott, 2005). He questioned the tendency to use teachers' beliefs as an explanatory principle for practice. The general affirmative answer to the question of a possible positive correlation between teachers' beliefs and the classroom practices appears to be a premise rather than a result of belief research. This is so in spite of occasional calls to look into another and less researched question of a possible opposite relation between practice and beliefs (e.g. Guskey, 1986). Skott's criticism, then, challenges the tradition in belief research that Wilson and Cooney later described as a basic assumption of research on teachers' beliefs (cf. the quotation above).

Also, a rather more substantive criticism of belief research has been raised, especially of the highly individual approach normally adopted. This is related to another dimension of the concept of beliefs, i.e. the one of their relative stability, not only in the sense of being unsusceptible to change or develop in the course of time, but also in being stable across contexts. The first of these – stability over time – is well substantiated, although it is generally agreed that beliefs may change for instance in the course of reform oriented teacher education programmes that model the type of teaching envisaged and that include collaborative efforts on the part of the participants (Wilson and Cooney, 2001). The other sense – stability across contexts – is more problematic and concerns the primarily individual or social character of beliefs. Mainstream belief research has assumed that beliefs are individual constructs that are indeed stable across contexts.[1]

Hoyles (1992) and Lerman (2001; 2002) both argue that the very idea of a context-independent mental construct of belief misrepresents the social character of human functioning. Hoyles claims that one should acknowledge the situated character of beliefs, and Lerman (2001) phrased his point like this:

> I want to suggest that whilst there may be a family resemblance between concepts, beliefs and actions in one context and those in another, they are qualitatively different by virtue of those contexts. (p. 36.)

Skott (2001) also claimed that a more social perspective is needed in belief research, one that acknowledges that the objects and motives of the teacher's activity emerge from the interactions with specific students in the specific classroom. Further developing the argument, he pointed to the teacher's involvement in multiple, simultaneous communities of practice each of which frame certain aspects of the teacher's activity, and claimed that beliefs of mathematics and its teaching and learning played variable roles in different contexts dominated by adherence to each of these of communities (Skott, 2002).

The two sets of criticism raised above – one primarily methodological, the other primarily substantial – are interconnected and between them they point to a certain irony. On the one hand, belief research may be seen as fuelled by increasingly social emphases in the theory of mathematics education, while on the other it appears – with very few exceptions – to be conducted from an overly individual perspective of an autonomous teacher determining what gets taught and learnt in mathematics classrooms. Also, the two sets of criticisms share a concern for the risk of the researcher creating or assuming the existence of an unambiguous object of study (students' or teachers' beliefs) that may be alien to

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[1] This is often not made explicit, but can be inferred from what has become a dominant methodological approach that would otherwise lose its credibility. If, for instance, teachers are not able to 'carry' beliefs with them across contexts it would hardly make sense to look for compatibility between beliefs espoused in interviews or questionnaires and those inferred from the practices of their mathematics classrooms.

the students and teachers in question and the significance of which is presumed rather than investigated. In chapter 3, we shall elaborate on more specific aspects of these criticisms and describe ways in which we have tried to take them into consideration in the KappAbel study.

One further aspect of belief research needs to be addressed, namely the lack of terminological clarity. There have been many attempts to distinguish between beliefs, conceptions, attitudes, world views and several other phrases all of which are meant to capture significant aspects of students' and teachers' meta-mathematical orientations, including those related to mathematics teaching and learning. We agree with Wilson and Cooney (2002, p. 145) that continued scrutiny of these terms and of the possible relationships between them is unlikely to contribute substantially to increased understandings of the roles of teachers' and students' school mathematical priorities for the practices unfolding in mathematics classrooms. In spite of that, we find the conceptual and terminological discussion useful to indicate the emphases of the present study.

This terminological discussion adds yet another dimension of the notion of beliefs to the two ones discussed in relation to the criticism of belief research above. This dimension concerns the emotive or cognitive character of beliefs.

Beliefs   Attitudes   Emotions
<-->
Stability   Intensity

Figure 2.1 Spectrum of Types of Affect
Source: McLeod (1992)

According to McLeod's review (1992) of research on affect in mathematics education, "beliefs", "attitudes", and "emotions" were used to describe a wide range of affective responses to mathematics. These three terms are not easyly distinguishable but they vary in stability: beliefs and attitudes are generally stable, but emotions may change rapidly. They also vary in level of intensity of the affects that they describe, increasing in intensity from "cold" beliefs about mathematics to "cool" attitudes related to liking or disliking mathematics to "hot" emotional reactions to the frustrations of solving non-routine problems. McLeod also distinguishes beliefs, attitudes, and emotions to the degree to which cognition plays a role, and the time they take to develop. In figure 2.1, affect in a broad sense in mathematics education is positioned along a spectrum that runs from *stability* and “cool” on the left (the cognitive end of the spectrum), to fluidity and *intensity* on the right (the affective end of the spectrum).

This is not the only analytical description in different dimensions of the affective area in mathematics education research (cf. Evans, 2000:43-45) but, in terms of terminological clarification, we find these three aspects and the interrelated characteristics both operational and meaningful. This figure illustrates a continuum from cognitive to affective aspects of people’s relationships with mathematics. Besides the teachers’ beliefs as presented above, *beliefs* include self-perception (e.g. “Mathematics - that’s what I can’t do.” (Wedege, 2002)), aspects of identity (e.g. “In my life, I will never need any mathematics.”), and confidence; and *attitudes* (e.g. “The importance of mathematics is increasing with technological development in society.” or “Mathematics is the most terrifying school subject.”) are more stable than *emotions* (e.g. panic or joy).

In the first description of the KappAbel study, “attitudes towards mathematics” was understood to broadly include people’s affective relationship with mathematics comprising the three aspects: beliefs, attitudes and emotions. From the above discussion, it follows that ”attitudes”, the term used in the working title of the present study, is often conceived of as including beliefs and emotions with its position in the middle of the spectrum. For the purposes of the present study we shall use the term slightly differently and change our terminology. Our intention is not to locate ourselves in the middle of the cognitive-affective spectrum. In the McCleod terminology, we study students’ and teachers’ beliefs of and attitudes towards mathematics at the cognitive end of spectrum, and we name these phenomena peoples’ *views of mathematics*.

2.2 Practices

The notion of practice has come to play a different role in mathematics education over the last decades. This follows the development hinted at above towards still stronger social perspectives on mathematics teaching and learning. One aspect of this ”social turn” in mathematics education (Lerman, 2000) draws on the work of Lave and Wenger (Lave, 1988; Lave and Wenger, 1991; Wenger, 1998). It views learning not as a separate activity, but as the learner’s development of increasingly sophisticated ways of taking part in the practices of the community in question. Another aspect is what has come to be known as an emergent perspective on mathematics classrooms (Cobb and Bauersfeld, 1995; Cobb, Yackel, McClain and Whitenack, 1997). It draws on both symbolic interactionism (Blumer, 1969) and socio-cultural theory (Vygotsky,1978; Vygotsky, 1986; Wertsch, 1985) to challenge exclusively cognitive views on knowing and learning. A discussion of the relationships between these and other aspects of the social turn is beyond the scope of this report. For the present purposes it suffices to note that it is a common feature that they challenge the dominant metaphor for learning as acquisition and supplement or replace it by the one of participation (e.g. Sfard, 2003).

This participatory view of learning, however, is not in and by itself a recommendation for how to teach. Rather, it may be seen as an orienting framework and as an analytical tool to make sense of the possibilities for student learning. In other terms, students always participate in classroom practices, even when they are apparently not involved in activities that may be deemed mathematical. From this perspective, ”studenting”, i.e. manoeuvring as a student in relation to the perceived demands in the classroom in question, is sometimes a much more dominant activity within the classroom practices than doing mathematics. Student learning, then, must be seen as participation in a classroom practice that is jointly constituted by teacher and students in that particular classroom within the broader social context of the school and of society at large. That, of course, may to a greater or lesser extent encompass elements that may be thought of as mathematical. And in turn the analysis may suggest ways of inserting more elements of what is deemed as valuable mathematical activities in the classroom in question.

It follows from what has been just said that there is a need to question the common notion of ”the teacher’s practice”. Of course, if ”practice” is taken in just the very narrow sense of observable behaviour, this notion does make sense. But understood in the broader sense of the social practices unfolding in the classroom, any connotation that it ”belongs” to the teacher is a contradiction in terms. This does not mean that the teacher may not be an extremely significant participant in that practice, and in some sense seen as the local master, who officially legitimates the character of the trade, i.e. of mathematics. However, it is somewhat ironic that the possessive connotations of the phrase of ”the teacher’s practice” have not been reduced with the increasing social emphases in mathematics education research. On the contrary, the teacher is to an even greater extent expected to take on a role of autonomous decision maker who by virtue of her continuous reflections on the interactions controls the learning opportunities of the classroom. This is so not least in much belief research focusing on belief-practice relationships. These often end up with condemning descriptions of teacher inconsistency as the main explanation of why – from an observer's perspective – classrooms do not develop according to the espoused priorities of the teacher (e.g. Raymond, 1997). The explicit or implicit assumption is that it is indeed the teacher's practice.

The notion of the teacher owning the practices in the classroom, then, does not go well with the concept of practice outlined above. Practices are social and constituted interactively within the broader social framework in which it is situated. Also, they are structured by the engagement of the participants in multiple, simultaneous activities, mutually constraining and supporting each other. In Lave's terms (1988), these activities act as the structuring resources for participation in the practice in question. She gives the example of people doing mathematics and shopping at the same time, for instance when looking for best buys. The character of the mathematics you do depends on the shopping, as you may for instance rely on methods that are only vaguely related to those you were taught in school. But the character of the activity of shopping also depends on if and how mathematics becomes part of it. Shopping, then, is no longer the same activity, or at least it takes on a different flavour, if doing calculations to compare the unit price of different brands of cheese enters the picture. One may add that pleasing your children with what you buy, hurrying in order to avoid getting a parking ticket, and quarrelling with your wife on the phone are other activities that may enter the picture as asymmetrical structuring resources for participation in the practice related to the arena of the supermarket.

Valero builds on Lave's notion of structuring resources when describing the practice of mathematics teaching in three different schools on three continents (Valero, 2002, p. 278 ff.). She finds that the activities that mutually structure the practice of teaching in the three schools are differentially related, not least because of the differences between the broader social contexts in which the schools are situated.

Zooming in on the students, one may expect that especially what we called the activity of studenting is also framed by the broader social setting, and that this activity is reciprocally structuring activities some of which are more closely related to the learning of mathematics or to mathematics itself. These other activities may include preparing for the test, practising pre-given procedures, generalising patterns, investigating concepts and definitions, looking for a formula that will provide an answer to a task one is trying to solve, and many more.

The ”practices” that are referred to in the title of this study are the ones of mathematics classrooms in classes that participate in the KappAbel competition. The previous discussion of the notion of practice indicates that our notion of *mathematics classroom practices* is not one that equates teaching practice with observable teacher behaviour. Also, the use of the plural when talking about practices does not only suggest that what happens in different classrooms is not necessarily the same. Rather, we take it to mean also that practices within each classroom may be considered as continually regenerated and further developed by the participants' involvement in multiple simultaneous activities that mutually structure and frame each other to constitute the practice of the classroom in question. This understanding relates and feeds back into the discussion of the views of teachers and students of mathematics and its teaching and learning.

2.3 “Didactical contract” as a metaphor in the study

The interplay – or the social and mathematical interaction – between teacher and students within the frame of the mathematical instruction is crucial in this study, where the topic is the potential change of practices of mathematics classrooms and in the teacher's and students' views of mathematics. We found it relevant to involve the metaphor of ”didactical contract” in the design of the study and in subsequent analysis because it might combine the emerging school mathematical practices with views of mathematics and of the learning of mathematics. If you want to infer whether views and practices have changed, you need to look beyond the immediately observable actions and organisations of classroom activity, e.g. beyond whether more or less time is spent on group work. You have to study if and how the mutual expectations of the participants in those practices have changed, i.e. if and how the contract regulating their interactively developed contributions is evolving.

Brousseau's concept of didactical contract is well known, or at least the term is frequently used. It originates in the framework of the French school of ”Didactique des Mathematiques”, and for Brousseau it is inextricably linked to the theory of didactical situations (Brousseau, 1986). As such the didactical contract is formed by the task and its intention, both of which are initially defined by the teacher. Brousseau phrases the connection between the didactical contract and the didactical situation like this:

> The didactical contract is the rule of the game and the strategy of the didactical situation. It is the justification that the teacher has for presenting the situation. But the evolution of the situation modifies the contract, which then allows new situations to occur. (Brousseau, 1997, p. 31)

Adopting a somewhat different perspective, Balacheff links the didactical contract to the norms for social interaction in a broader sense. To some extent, then, he removes the concept of the didactical contract from the theoretical framework and the empirical studies on which it is originally based, and he defines the didactical contract as follows:

> The rules of social interaction in the mathematics classroom include such issues as the legitimacy of the problem, its connection with the current classroom activity, and the responsibilities of both the teacher and pupils with respect to what constitutes a solution or to what is true. We call this set of rules a *didactical contract*. A rule belongs to the set, if it plays a role in the pupils’ understanding of the related problem and thus in the constitution of the knowledge they construct. (Balacheff, 1990:260).

In mathematics education literature outside France, the notion of didactical contract is more often used in this latter, broader sense than in the one closely connected to the theory of didactical situations. This is the use we shall make of the term in the following as well. Thus, our use of the didactical contract does not imply that we import the general theoretical framework of the theory of didactical situations. Rather, we use the term *didactical contract* as a metaphor for the set of implicit and explicit rules of social and mathematical interaction in a particular classroom. The didactical contract, then, in our terminology constitutes the rules of the game in that classroom, rules that on the one hand frame the practices that emerge and on the other are regenerated and transformed by those very same practices.

Among the rules of a didactical contract concern//? three central issues may be addressed:

1. What is mathematics and mathematics education?
2. How do you learn mathematics?
3. Why do you learn mathematics?

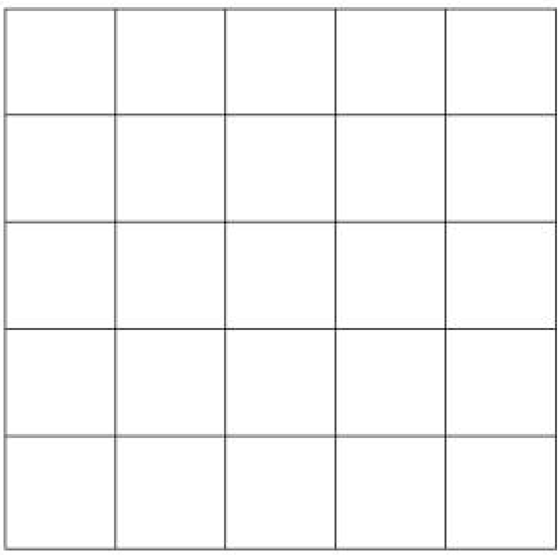
The following statements on mathematical problem solving are elements taken from contracts supporting different types of practices:

- The students challenge each other with ”mathematical nuts” at least once a week.

- The teacher goes through the theory necessary for the students to solve the problems.
- The students collaborate in small groups to solve the problems.
- Problem solving is only a matter of practising the skills.
- Problem solving is primarily use to work with mathematical concepts.
- The students can always find an example in the textbook corresponding to the problem they have to solve.
- Problem solving requires mathematical imagination and creativity.
- The students should not let the task context disturb because this is only a pretext for using mathematics.
- The problem text refers to experiences from the students’ daily life, which should be actively involved in the problem solving.
- The students individually hand in exercises once a week and the teacher corrects them.
- The students hand in exercises in groups once every two weeks, and another group corrects them under the teacher’s supervision.

Figure 2.2 Problem from the national final in Norway 2004/05

Problem 2
One in every row, column and diagonal
Place the five pieces in the 5x5 square in a way that you never have two pieces in the same row, column or diagonal.
Find as many different solutions as possible:



The problems in the qualifying rounds of KappAbel are not consistent with a didactical contract in which every student who has read and understood the theory, gone over the examples in the textbook and solved the exercises is expected to be able to solve the problem. Problem number 2 from the national final in Norway 2004/05 may be seen as examplary (see figure 2.2). This task seems to require the students to engage in a sort of systematically creative investigation, not supported by contract just described.

2.4 Changing views and practices

It follows from the discussion in the previous sections that we do not expect teachers' views to change easily. Also, following some of the criticism of mainstream belief research, we do not expect teachers' views to be immediately and uni-directionally related to his or her contributions to the classroom interactions, let alone to the practices of mathematics classrooms more generally. However, views and practices do change and so do didactical contracts as a symptom of changing practices and views of mathematics. The question we are addressing in this study is if and how the KappAbel competition may facilitate such change.

One of the differences between the KappAbel competition and most other initiatives towards reforming mathematics education is the character and sequence of the steps expected to bring about the envisaged changes. Most attempts to bring about change in teachers' views on mathematics and its teaching and learning go via a pre- or in-service teacher education course. Based on the premise of beliefs being a "significant determiner" (cf. the quotation from Wilson and Cooney p. 33) of the teaching-learning practices in mathematics classrooms, teachers are then expected to carry their newly established, reformist educational priorities into the schools (see figure 2.3).

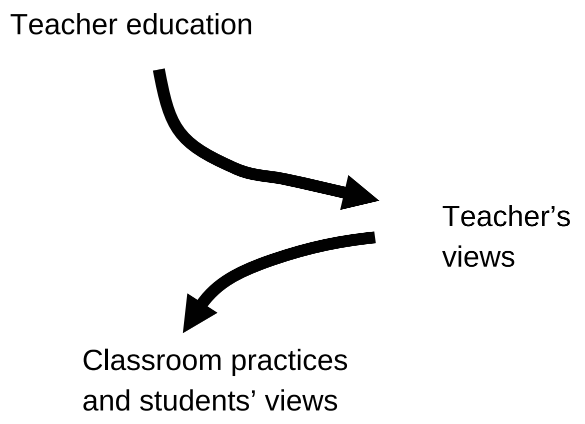


Figure 2.3

In KappAbel the intention is to directly restructure the practices of mathematics classrooms. Phrased negatively, it does not seek first to influence the teachers' views of mathematics and its teaching and learning and subsequently to expect the teachers to carry envisaged changes in teaching-learning practices into the schools. Instead KappAbel aims to transform the classroom practices by inserting new types of tasks and novel ways collaborating directly into mathematics classrooms (see figure 2.4).

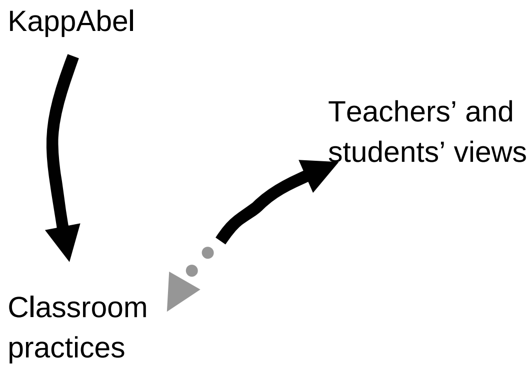


Figure 2.4

Using the notion of practices that was described above to interpret the situation, KappAbel may be expected to contribute to changes in the practices of mathematics classrooms, because (some of) the activities that students and teachers become involved in are fuelled by tasks and collaborative processes that are radically different from those that dominate the traditions of the school subject and teachers' school mathematical priorities. This is most probably not a comprehensive transformation of the overall classroom practices. Rather it may be, that for instance the activity of practising pre-given procedures takes on a less prominent position within the practice, or that the one of preparing for the test is transformed, because the test now consists of rounds of the KappAbel competition or is structured to prepare the students for participating in those rounds.

Classroom practices may change. Change, however, is not merely a question of implementing a few new ideas, for instance in the form of tasks that are meant to insert collaborative and investigative elements in mathematics instruction. Change is a matter of teachers and students engaging differently in the activities that mutually constrain and support each other so as to constantly regenerate and further develop the practices of the classroom. It is in this sense that we may expect KappAbel to initiate changes in the practices of mathematics classrooms. As a consequence of their involvement in these changing practices and of their renegotiation of the didactical contract, students and teachers may develop new ways of conceiving of mathematics, of mathematics in schools, and of the teaching and learning of mathematics, i.e. they may develop new views.

Chapter 3

Research design

In the KappAbel study, we have tried to keep the purpose of the study, the theory and conceptual framework, the research questions, the methods and the sampling strategy in balance (see Robson, 2002). We had a very tight schedule caused by the KappAbel competition it self (first and second round, project work, semi-final) and the limited resources in terms of time and money. Like in many other studies we had to adapt and modify our plans and sampling strategy through the whole period. But possibly to a greater extent than in most research we were under severe constraints, because the KappAbel schedule allowed for little flexibility in terms of timing of the different parts of the study.

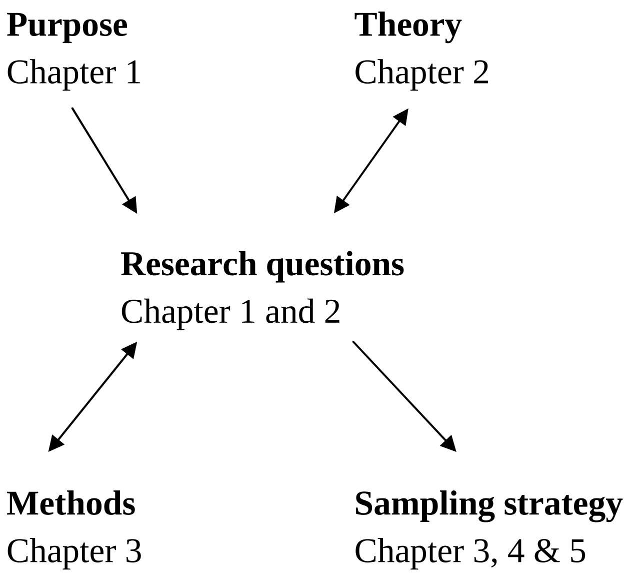


Figure 2.1 Framework for research design

The framework presented in figure 2.1 illustrates the design process in the KappAbel study. The purpose of the research study as formulated by the Nordic Contact Committee and adapted by the research team is presented in chapter 1. This has been our point of departure and guide through the study. The research questions are presented in chapter 1 and given a more explicit formulation in chapter 2. As suggested in the figure, we find the research questions, which have been specified and adapted through the study, decisive in the design process. The theory and terminological framework are presented in chapter 2. This framework has been discussed and developed in tandem and continuous interplay with the empirical investigations. In chapter 3, our methods are presented and discussed together with some methodological issues. The quantitative and qualitative methods in the study are inter-related and balanced with purpose, theory and research. The sampling strategy is presented and discussed in chapter 3, 4 and 5. Several times through the study we had to change the strategy and decide upon where, when and from whom we should seek the data.

Our approach in the design process has been rather pragmatic in that we mix methods from the post positivist research paradigm, the constructivist paradigm and the transformative paradigm (See Mertens, 2005). However, our epistemological stand point is transformative i.e. we recognize that there is an interactive link between researcher and participants, - that knowledge is socially and historically situated.

The methodological challenges and difficulties related to research on teachers' and students' views of mathematics and their mathematics classroom practices, which are mentioned below, implied that we had to use different empirical approaches to the problem field. The KappAbel research project includes five types of empirical data – quantitative as well as qualitative (see the time schedule in table 3.1). These are

- TS1: A questionnaire administered to the teachers of 2856 grade 9 mathematics classes in the 2004-2005 academic year, in Norway.
- TS2: (a) A short questionnaire sent by email to 351 teachers whose classes took part in the two introductory rounds of KappAbel. (b) A questionnaire administered to 15 of these teachers whose classes intended to continue with the project work, whether their classes progressed to the semi-finals or not.
- Interviews conducted with five teachers and three groups of students at the national semi-finals; further interviews with two teachers and two groups of students.
- The reports and process log books of seven classes on the project work of *Mathematics and the Human Body*. (Students from these seven classes were among those interviewed or observed).

- Observations of 10 lessons in 3 classes at a school with strong traditions of participation in KappAbel. One of the classes progressed to the national semi-finals.

The findings from the first teacher survey (TS1) were implemented in the design of the questionnaire TS2 and of the interview guides (see chapter 5.2).

Table 3.1 Time schedule of data collection in the KappAbel study

December 2004	Teacher survey 1 (TS1)
February 2005	Getting in contact (email survey)
March - April 2005	Teacher survey 2 (TS2)
April 2005	Interviews with teachers and students
November 2005	Case study

Chapter 3 reports on the design and methodology of the KappAbel study. We present and discuss some methodological difficulties related to the purpose of this particular study and some general difficulties. Then we present the quantitative study in the first teacher survey (TS1) and the qualitative studies in the second teacher survey (TS2), the interviews, the observations and the document analysis. Finally we report on ethics in the study related to anonymity, information to and involvement by the participants.

3.1 Methodological difficulties

In chapter 2 we discussed, and to some extent challenged, what we consider dominant approaches to research on teachers' views of mathematics. Furthermore we presented the theoretical framework adapted in our empirical investigations. In this passage, we present our main methodological considerations in the KappAbel study.

In the paper "Researching potentials for change: the case of the KappAbel competition" (Skott and Wedege, 2005), we addressed three interconnected specific methodological difficulties in the study: (1) the problem of not using conceptual frameworks on mathematics that are well grounded empirically; (2) the question of overemphasising teachers' views of mathematics for their educational decision making; (3) the problems that no terminology carries unequivocal meanings and that more than an indication of agreement or disagreement with the rhetoric of reform is needed to outline teachers' and students' school mathematical priorities. Furthermore, in relation to (1) and (3), we addressed a general methodological issue concerning elements of power relationship in any research interview and survey.

The two sets of criticisms towards mainstream belief research presented in chapter 2 – one primarily methodological, the other primarily substantial – are interconnected and share a concern for the risk of creating or assuming the existence of an unambiguous object of study (students' or teachers' views) that may be alien to the students and teachers in question and the significance of which is presumed rather than investigated. We shall elaborate on more specific aspects of these criticisms below and, in the following sections, describe ways in which we have tried to take them into consideration within the time frame available for the KappAbel study.

It is apparent from the introductory description of KappAbel that the competition seeks to influence the immediate teaching-learning practices of the participating students and teachers as well as their views of mathematics. It does so by suggesting reformist classroom processes that are not necessarily in line with those that normally dominate the classrooms in question or with the teacher's general school mathematical priorities (see chapter 1.5 and 2.4). This is different from attempts to develop teachers' views for instance through pre- and in-service teacher education. In the latter cases, the expectation is often that a change in an independent mental construct of views will subsequently inform teaching-learning practices, as the teacher 'carries' her reformed attitudes to school mathematics into the different setting of the mathematics classroom. In contrast to this, KappAbel seeks to have an immediate impact by inserting a different set of teaching-learning practices into that setting by structuring the prevailing type of collaboration and by determining the types of task to be used. This is the first of KappAbel's intentions as described in chapter 1. The other intention may be rephrased as an attempt to capitalise on the part of the reciprocal view-practice relationship that has so far been researched the least, i.e. the one from practice to views. In these terms the intention is to influence teachers' and students views through an imposed set of changes in collaborative structure and types of task.

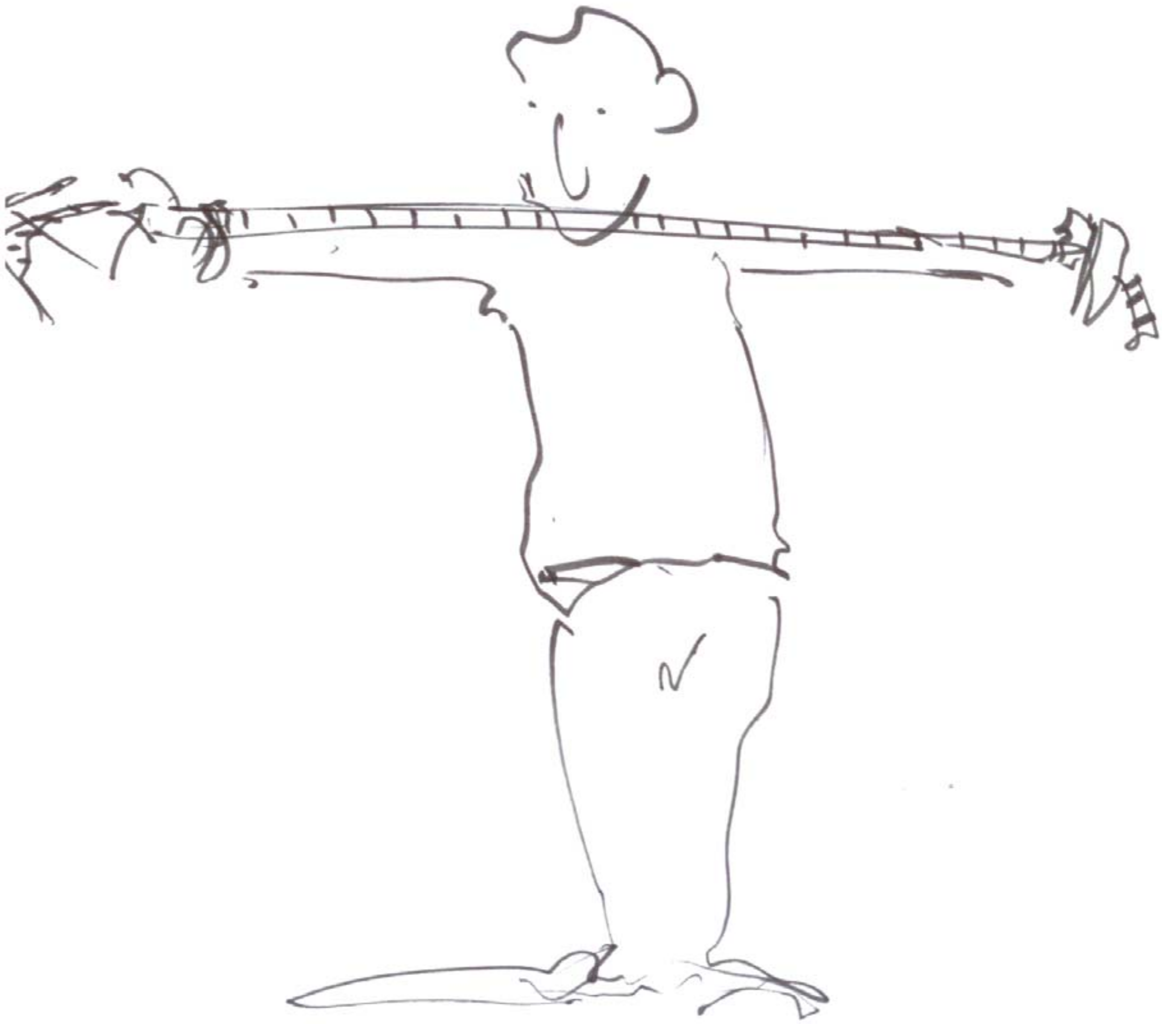
One aim of the present study is to shed some light on the extent to which KappAbel is successful in this: Does it succeed in enrolling teachers and students in new mathematical practices through the provision of new resources for those practices in the form of becoming engaged in the competition and the tasks? And if that is the case, does enrolment in such practices – initiated by outsiders to the specific classroom – pave the way for changes in the way teachers and students conceive of mathematics and its teaching and learning?

The design of the study seeks to take into account the critical remarks made above about mainstream belief research. In the initial project description, we questioned an assumed straight-forward relationship between teachers' views and the classroom practices. Inspired by Pehkonen and Törner (2004), we also pointed to the need to distinguish between teachers' ideal and real teaching of mathematics in questionnaires and interviews. Finally, and referring to Wedege and Henningsen (2002), we pointed to a possible conflict between the teachers' and students' views of mathematics expressed in their own words and the researchers' words expressed in a questionnaire. Later in the process of designing the study, we considered in particular three specific, interconnected methodological problems that may be seen as instances of the general criticisms mentioned in chapter 2.

First, there has been a tendency in belief research to build on Ernest's (1989, 1991) distinction between three educationally relevant views of mathematics. Ernest claimed that the subject may be seen as (1) a set of unrelated facts and procedures, a toolkit that is useful for purposes external to the subject itself; (2) a Platonic and objectively existing body of knowledge to be discovered; and (3) a problem-driven and process oriented dynamic field, an ever-expanding human creation. Pehkonen and Törner (2004) is one example of a use of Ernest's scheme. Phrasing the three perspectives as a toolbox view, a systems view and a process view of mathematics, they asked teachers to locate their real and ideal teaching practices on an equilateral triangle with the three views placed at the vertices.

Ernest's trichotomy was made with reference to the apparent philosophical relevance of the three views and to what he terms their "occurrence in the teaching of mathematics" (1989, p. 250). Further, Ernest links the three views of mathematics immediately to corresponding views of the role of the teacher and the student in mathematics classrooms.

Notwithstanding the significance of Ernest's scheme as a way of structuring the educational discourse on mathematics, it may be questioned whether the three views and the corresponding connections to teaching and learning are well grounded empirically. If this is not the case, it is a problem to the extent that the scheme determines the types of questions asked and the types of answers obtained in research. In other terms, it may be argued that Ernest's scheme does necessarily capture significant aspects of the teachers' school mathematical priorities, but impose the trichotomy (toolbox/system/process) on the teachers or students in question, if it is used as an analytical tool.



Second, it is not obvious how important the teachers' views of mathematics are, even if the tripartite scheme does capture significant aspects of it. Even if the mathematical priorities of a teacher may be seen to be in line with one or a combination of the three views of mathematics mentioned, it does not imply that such a view is of particular significance neither to way the teacher views him- or herself as a teacher of mathematics, nor to the way he or she contributes to the interactions of the mathematics classroom. Consider, for instance, a teacher with a strongly child-centred view of education. It may be that she also considers mathematics to be associated with a certain combination of the three elements of the scheme and that she is therefore able to position herself in the equilateral triangle suggested by Pehkonen and Törner. In and by itself, however, this does not indicate how significant her mathematical priorities are for her contributions to the interactions in the classroom.

This problem relates to the well-known question of the centrality or peripheral character of beliefs. However, in contrast to most studies in mathematics education it contextualises this question by acknowledging that the objects and motives that dominate the teacher's activity in a mathematics classroom may not be the same as the ones that frame her activity when being an object of study. In other terms, structuring a questionnaire, an interview protocol or an observation schedule along the lines of the Ernest scheme may reify the view of mathematics and attach a much greater significance to it than what is warranted if a more grounded approach to research on teachers' and students' educational priorities and activity is used.

Third, no terminology carries unequivocal meanings. This is so also for the rhetoric of the reform in mathematics education. More specifically, an emphasis on notions of problem solving and project work, or claims that school mathematics should be directed at working with the students' real world problems do not necessarily mean the same for all. This is banal, but it challenges the use of fairly closed questionnaires or interview protocols as a means of accessing teachers' or students' school mathematical priorities. At the same time it may explain the prominence of the approach that Lester described (cf. chapter 2) of becoming involved in a circular argument: we do need more than espoused views of mathematics and its teaching and learning in order to warrant a claim about the existence and character of such views. From this perspective Lester's point may be rephrased to say that although several approaches may be needed in the study of beliefs, these approaches do not necessarily shed light on the same object – a context-independent mental construct called beliefs. In spite of that, multiple methods have proved to be of relevance also to overcome the problem of the multiple meanings of the terminology used. For instance, classroom observations with follow-up interviews stimulated by recordings of interactions in the classroom in question have been used successfully. The strength of this combination, however, is not that it draws a more accurate picture of a stable and de-contextualised construct of school-related mathematical views. Rather, it is that it may situate teachers' and students’ reflections about mathematics and its teaching and learning within the context of a relevant classroom and contribute to an understanding of how the teacher interprets the interaction in question from the perspectives that the teacher herself finds the most significant.

Symbolic violence

One further aspect of our methodological considerations - related to all the three problems discussed above – is related to Bourdieu’s notion of symbolic violence. In the process of research design, and as a response to the methodological problems outline above, we concluded with a wish of using conceptual frameworks that are well grounded empirically, as far as possible. During this process, we came across a discussion by Bourdieu (1993) on methodical questions related to surveys and interviews. He introduced a notion of “symbolic violence” which fitted in with our preliminary considerations on the researcher’s imposing theoretical frameworks on teachers. As a sociologist, Bourdieu is well known within mathematics education mainly because of his theoretical concepts *cultural capital* and *habitus* developed in tandem with his empirical – mainly quantitative – research (see for example Wedege, 1999).

In his last major research project, *La misére du monde* (“The Misery of the World”) Bourdieu and his team (1993) collected and analysed testimonies from hundreds of respondents about their lives. In a retrospective methodological chapter called *Comprendre* (“To understand”), Bourdieu presents the underlying epistemological assumptions of the diverse operations of the inquiry: selection of interviewers, transcription and analysis of the interviews, etc.: ”When reading our texts, the reader will be able to reproduce the work of construction and understanding of which they are produced.” (Bourdieu, 1993, p.1390, our translation).

The research interview, Bourdieu claims, is a social relation with effects on the results obtained. This is so although the research interview differs from most ordinary dialogues in that it aims at pure cognition and by definition of scientific inquiry excludes the intention of using any form of symbolic violence that may influence the responses. However all kinds of distortions are inscribed in the mere relation of an inquiry, for example in an interview. It is, for instance, the interviewer who initiates and sets the rules of the game. This creates an asymmetry between interviewer and interviewee, the significance of which is doubled by a social asymmetry, if there is a hierarchical relationship between them in terms of capital, especially cultural capital. In any piece of research, this inserts a power relationship – an element of *symbolic violence* – that necessarily influences the results.

We may find a symptom of what Bourdieu would call symbolic violence in the study on mathematics teachers' attitudes towards mathematics mentioned above (Wedege and Henningsen, 2002). The authors did not find empirical evidence of the analytical value variables they used (e.g. abstraction, control, progress, democracy) in the teachers' own narratives, although between 58 and 98 per cent agreed upon these values in the questionnaire. There was, then, an apparent conflict between the teachers' conceptions of mathematics expressed in their own words and in the researchers' words in the value chart.

In the KappAbel study, we have tried to reduce this kind of symbolic violence in different ways. When constructing the questionnaires and the interview guides we tried not to impose theoretical constructs on the teachers e.g. we only used the three categories toolbox/system/process as theoretical background. Furthermore, we opened up to differences between real and ideal teaching like Pekhonen and Törner (2004). When the research assistant, Kjersti Wæge, who is a young woman with a mathematics teacher background, talks with other young mathematics teachers in their own language (Norwegian) this may help in reducing the distance in the dialogue and the significance of a hierarchical relationship between researcher and teacher.

3.2 The quantitative study – Teacher survey 1

The teacher survey (TS1) has a population of all Norwegian teachers in mathematics at 9^{th} grade in the academic year 2004/05, i.e. the same target group as the KappAbel contest in mathematics. One may find a technical report on data collection and handling in the first teacher survey in appendix B. The overall purpose of the teacher survey was to put into perspective the qualitative data from the small and heterogeneous material (questionnaire, interviews and observation) by means of a big quantitative data material. The intentions behind TS1 are to get some understanding of who the participating teachers are, both with regard to their objective characteristics (education, teaching experience, sex, etc.); to why they and their students participate in KappAbel; and to whether they consider themselves in line with the rhetoric of the current reform.

The latter of these intentions is to contribute with knowledge about the extent to which KappAbel is confirmative rather than formative. The question, then, is whether the participating teachers consider themselves more in line with the overall intentions of the competition than their non-participating colleagues. Considering the methodological problems mentioned above, the aim is not to point to any substantial 'objective' correspondence between the KappAbel intentions and those of the participating teachers, let alone to imply that the teaching-learning practices of the participating students and teachers are more or less in line with current reform initiatives than those of their peers. TS1 merely attempts to find if and how the terminology of the KappAbel project resonates with the priorities of the participating teachers to a greater extent than with those who do not participate. Furthermore, the statistical data from the large quantitative data set, TS1, was also used to see the qualitative data from a limited and heterogeneous sample (TS2 and the interviews) in a proper perspective: to what extent are the participants in the qualitative study different from the population in general in terms of their objective characteristics. For this reason the factual information obtained in the two questionnaires (the teacher's background and experience) is identical.

Design of the questionnaire

As mentioned above, the intentions of the teacher survey were to get some understanding of who the participating teachers are, both with regard to their objective characteristics (education, teaching experience, sex, etc.); to why they and their students participate in KappAbel; and to whether they consider themselves in line with the rhetoric of the current reform in mathematics education.

The questionnaire TS1 contains 45 closed questions and opens a possibility to write comments at the end (see the questionnaire in appendix 1.1). The questions are grouped under three headings:

A. Background and experience: the teacher's gender, age, educational background, education in mathematics, teaching experience in mathematics, number of classes at 9th grade, location of the school, and the teacher's self-image.

B. On teaching and learning mathematics: the teacher's school mathematical priorities expressed in their ideas of students' activities and the purpose of theses activities in the mathematics classroom.

C. On KappAbel: the teacher's (lack of) participation in KappAbel in 2004/05, the reasons for their (lack of) participation, the teachers' opinion about the three principal ideas of KappAbel, and finally the teacher's earlier experience with KappAbel.

In the group A, we added the question of whether the teachers regarded themselves as mathematicians, mathematics teachers or as generalist teachers (question 8). This was based on the assumption that a teacher's professional self-image may be correlated to their view of mathematics as a toolbox, system or process.

The questionnaire was tested in a small pilot (19 teachers) during a conference for teachers on mathematics education in Trondheim, November 2004. On this background, we made a few linguistic changes of the questions.

3.3 The qualitative study

The qualitative study included

- a questionnaire administered to 15 of the teachers who participated in KappAbel in 2004/05 and whose classes intended to continue with the project work;
- interviews with five teachers and three groups of students at the national semi-finals; interviews with two other teachers whose classes had also participated in KappAbel and with one group of students from each class;
- document analysis of reports and process log books of seven classes on the project work of *Mathematics and the Human Body* (Students from these seven classes were among those interviewed or observed), and
- observations of 10 lessons in altogether 3 grade-10 classes at a school with a strong tradition for participation in KappAbel; all the classes had participated in last years KappAbel, and one of them had made it to the national semi-finals;
- Interviews with the teacher of the three classes mentioned above, and a group interview with students from the class that progressed to the semi-final.

In order to get in touch with the teachers who participated in KappAbel round 2, a short questionnaire was sent by email to 351 teachers whose classes took part in the two introductory rounds of KappAbel. Among these teachers were the teachers who had qualified for the semi-final. The questionnaire contained a question about the teachers' impression: Will your engagement in the KappAbel competition have an impact to the mathematics education in your class during the rest of the school year? And if so in what way: the type of problems, the way the students worked (e.g. project work), the amount and type of group work, the students' attitudes towards mathematics? Furthermore, we asked if the teacher and the class planned to do the project on "Mathematics and the Human Body". If the answer to the last question was in the affirmative, we asked for permission to contact the teacher again later.

The questionnaire

The second questionnaire, TS 2, was administered to 15 teachers whose classes took part in the project work on *Mathematics and the Human Body* following the two introductory rounds of KappAbel. The items of TS2 were to some extent informed by a preliminary analysis of the responses to TS1 (see the questionnaire in appendix 2.1). For example we used factor analysis in TS1 as a way of reducing the dimensionality of the problem (The teachers' school mathematical priorities) and find latent structures in the data (see appendix A). These structures were created solely from the data. The following dimensions were constructed on the basis of a factor analysis with three factors: (1) Patterns and investigations; (2) Structures and logical reasoning; (3) Tools and applications. The theoretical framework (process/system/ toolbox) was not involved in the analysis at this stage. However, from the outset, our questions in TS1 were constructed on the basis of these three theoretical categories. Consequently, the clear correspondence between the result of the factor analysis and framework is hardly surprising.

In TS2, some of the items are closed, but the limited number of respondents allowed us also to use more open and qualitative response items. Some of these are fairly traditional, and for instance ask the respondent to reflect on good and bad experiences with teaching mathematics. Other items require the respondent to comment on statements made by (imaginary) colleagues about key aspects of KappAbel (collaboration, project work, gender). One example reflecting a teacher's view of boys' and girls' work in the mathematics classroom reads like this: "Boys and girls have different strengths and weaknesses in mathematics, and should be allowed to cultivate these. It is therefore often an advantage if boys work with boys, and girls work with girls." (item 26.2)

The intentions behind this dual format of the open items may seem self-contradictory. On the one hand they aim to allow the respondent to focus on what he or she sees as the most significant aspects of mathematics teaching and learning. This is opposed to a situation in which they are exclusively to respond to items framed for instance by the scheme (toolbox/system/process). On the other hand, the other set of items invite the teacher to respond to a set of very specific statements from their imaginary colleagues. These may provoke a concrete response, but are meant to contextualise the teacher's reaction by involving him or her in a virtual dialogue that allows any response whatsoever. In spite of the concrete wording of these items, then, they intend to allow the teacher not only to signal for instance degree of agreement or disagreement, but to reject the agenda inherent in item altogether.

Interviews, observations and documents

Semi-structured interviews were conducted and taped with five of these teachers and three groups of students during the national semi-finals, and with two more later on, - in total seven teachers and five group of students. The intention was on the one hand to let the teachers and students talk freely about what they conceive as significant and valuable teaching/learning experiences without initially framing their answer within a pre-conceived scheme. On the other we wanted to get an impression of how they perceived KappAbel and their own participation in it in relation to their everyday teaching (see the interview guide in appendix 3.1). This may be seen as an attempt to strike a typical balance in semi-structured approaches to research between too tight and too lose a structure. For our purposes, and to minimize the symbolic violence in the interviews, we have tended to move towards the lose end of the continuum. The Norwegian research assistant, who has a background as a mathematics teacher, conducts the interviews, which may help in reducing the significance of a hierarchical relationship between researchers and teachers.

During the classroom observations two researchers were present, one doing the video recordings, the other making field notes, in part based on a preconceived observation schedule. The recordings primarily followed the teacher, who was also carrying an extra cordless microphone. The observation schedule recorded the structure of the lesson (how much time on whole class/small group/individual work), but was primarily concerned with

(1) the teacher: what types of questions are asked in whole class/small group/individual settings (e.g. open/closed)? what types of answers are given in those settings (e.g. results, conjectures, new problems); what types of responses provided (e.g. opening or closing further opportunities for investigation);
(2) students: are they on-task or off-task? Do they practice procedures and concepts? Are they involved in investigations? How do they respond to difficulties? What is the character of student-student communication?

At the beginning of the lessons notes were taken ion the physical appearance of the classroom, and after the lessons the types of materials used were recorded together with other possible comments of particular relevance to KappAbel.

The KappAbel project reports and logs from the seven classes involved in the study were collected as testimonies on the students' views of mathematics and they were analysed together with the interviews to get a better picture of the didactical contracts in the respective mathematical classrooms. We also intend using the interpretations of the individual teacher's response in the interview and to the questionnaire TS2 as well as the students' comments in the interviews as an interpretive device in relation to practice. This is not an attempt to make ”consistency” or ”inconsistency” descriptions of the teachers in question, but in some sense to triangulate our previous understandings (based on interviews and questionnaires) of what they prioritise in mathematics education and to get some indication of how prominently these priorities play in practice under the institutional constraints of classroom teaching. However, we do not consider this a case of traditional methodological triangulation. The problem is that methodological triangulation is normally considered an attempt to adopt a variety of different perspectives on the object under study. This is part of what we wish to achieve. However, some of the substantive criticism raised in chapter 2 of the lack of social orientation of mainstream belief research exactly questions the extent to which it is indeed the same object that is being studied in for instance a research interview and in classroom observations. It may be that teacher's views play a part for the classroom interactions, but the call for a more social approach in belief research is exactly built on the understanding that the teaching-learning practices unfolding in the mathematics classroom are not the teacher's practices in the possessive sense of that term. However, having teachers comment on specific interactions in follow-up interviews may provide increased insight into their school mathematical priorities, including what their potential role is in relation to unfolding classroom practices.

Ethics

In different letters to the participants in all parts of the study, we explained the research purpose, duration and procedures (see appendices 1.2, 1.3, 2.2, 2.3, 3.2, 3.4 and 3.5). We also informed about the kind of anonymity to be kept in the study. Here we distinguish between anonymity, which is the case in the teacher survey 1 (TS1), and confidentiality, which was the case in teacher survey 2 (TS2), the interviews with teachers and students, and the observations. By *anonymity* we mean that "no identifying information is attached to the data, and thus no one, not even the researcher, can trace back to the individual providing them." (Mertens, 2005, p. 333). By *confidentiality* we mean that "the privacy of individuals will be protected in that the data they provide will be handled and reported in such a way that they cannot be associated with them personally." (Ibid.).

In order not to influence the participants, we did not inform them directly in the questionnaire that we wanted to study the potentials for change of the KappAbel competition. However, in the information to the head masters and teachers, we called the study an "evaluation of the KappAbel mathematics competition". Although "evaluation" is not specified, our interest in the role of KappAbel is made clear. In the two letters to the students and to their parents, we mentioned that the interview, which was a part of the KappAbel study, concerned "mathematics education in school and the students' experiences" with it.

In relation to the students, we were concerned if we had to obtain approval from the parents (or their immediate superiors) – before the interview and observation, which would have made impossible the interviews during the KappAbel final. One of the normal general exemptions to approval of research conducted with school children is this:

> Research that is conducted in established or commonly accepted educational settings, involving normal education practices, such as instructional strategies or classroom management techniques. (Mertens, 2005, p. 333)

However, we contacted the Norwegian "Personvernombudet for forskning" (Research and the protection of privacy, www.nsd.uib.no/personvern) to be sure. The answer was that, when the study is about non-sensitive information ("like the learning of mathematics") related to 14 year olds information to the parents about the project and their passive approval is sufficient.

The taped interviews with teachers and students were transcribed in full. Before the analyses, these transcriptions were sent to the participants (teachers and students) for their potential objections or comments. Quotations from the interviews are to a certain extent corrected into written language in the report. The responsible persons in KappAbel have approved the passages from their interviews used in the report (chapter 1.4). The case study in chapter 5.6 was sent to the teacher for her comments before publication.

Conclusions

Research on views of mathematics is a difficult field, methodologically speaking. On the one hand, one needs a variety of different approaches to capture significant aspects of teachers' and students' views of mathematics. On the other one should not expect different methods to shed light on the same object of views, and consequently the idea of triangulation takes on a different meaning than one of locating a single point or object in a space of two or more dimensions by using a variety of such methods.

These methodological difficulties lead to some rather more conceptual ones: Does the constructs of views point to an object that is as stable and de-contextualised as one is sometimes led to believe? More specifically, are the claimed manifestations of teachers' and students' views of mathematics sufficiently grounded empirically to claim that they are part of the way in which the person in question conceives the world and him or herself within it? Does the significance attached to students' and teachers' views about mathematics resemble their own priorities, for instance in comparison with their general view of children, of school, of education and of mathematics in schools - or does that significance to a greater extent impose a mathematical perspective on the teachers and students, a perspective that may be much less important to them than to the researcher?

These questions may be summed up as one of the extent to which the researcher's imposition of the construct of views on the students and teachers in question is an exertion of symbolic violence in the sense of Bourdieu. And this feeds back into the methodological considerations: Is there any way of reducing the significance of that violence by exerting it differently?

In the KappAbel study we use a well-known methodological cocktail of large scale questionnaires, interviews, observations and analyses of students' work. Within the timeframe of the present study we have only managed to a very limited extent to come up suggestions for methods that break with or supplement the ones that are used in most other studies in the field. One may then rightly ask if and to what extent we fall prey to the same types of criticism that we raised against other studies in the field. To some extent this is undoubtedly so. However, our answer to that question has two more elements.

First, we do use well-known methodological tools, but we do so in ways that are not quite so well-known. For instance in TS 2, we seek to establish a context for teacher reflection by simulating an interaction between the respondent and a fellow-teacher. This intends to limit the extent to which the teacher conceives of the situation as a confrontation with an agenda imposed on him or her by the researcher. The intention is, then, to initiate a type of virtual dialogue, by inviting the teacher to react to a statement made by an imaginary colleague. We have also reasons to believe that the technical dimension of the methodological construction is important when it comes to evaluating the findings. As a consequence, the interviewer is a research assistant who is a mathematics teacher herself: young Norwegian mathematics teacher interviews young or experienced Norwegian mathematics teachers. This reduces the asymmetry inherent in the social relation of the dialogue. This was not the case with the students who obviously felt ill at ease in the situation. As an experiment we planned to invite one of the students who participated at the KappAbel semi-final to interview the rest of her group. However - as pointed to by Bourdieu – a social and cultural resemblance between interviewer-interviewee may create other problems and we did not go ahead this idea.

Second, and more importantly, we try to make the sense of the data collected with those tools in ways that acknowledge the significance of the types of criticisms listed. This last element, then, has more to do with the analyses and interpretations of the data than with the character of the data-collecting tools themselves. For example, a teacher's response to the first questionnaire is not interpreted as an indication of ”real” compatibility between his or her thinking about school mathematics and the intentions of KappAbel. Rather it is seen merely as an indication of the teacher's possible compliance with the rhetoric of the reform, and therefore as an indication of the extent to which participants in KappAbel to greater extent than other teachers consider themselves in line with the dominant reform discourse. Also, the interpretations do acknowledge that the results are not grounded in an empirical study and that whatever answer we obtain in terms of teachers' priorities of school mathematics should be interpreted with this in mind: we have not managed to set up a design that build on the teachers' own thinking. What we may be able to do is to shed some light on how the teachers, given the alternatives inherent in the Ernest scheme, prioritise aspects of mathematics. In relation to the aims of the study, we can claim that if it is fair to suggest that the priorities of the KappAbel competition include a vision of mathematics as at least encompassing the process perspective, the study may shed some light on the extent to which teachers at the level of rhetoric orient themselves in the same direction, when faced with a limited range of alternatives.

Chapter 4

The teacher survey

Who participate in KappAbel? Why do they participate? If you want to spread some light on the potential for change in mathematical classroom practices after students' and teachers' participation in KappAbel, then it is necessary to know something about the background and the motive for participating in the competition. In chapter 2, we have presented and discussed our theoretical framework and at the same time unfolded the problem complex of the study. In chapter 3, we have presented considerations and principles for the design of the investigations, e.g. Teacher Survey 1 (TS1), and for the total research design in the KappAbel study. That is how the quantitative survey in TS1 is interrelated with and works as background to the more qualitative investigations (more open questionnaires, interviews, class room observations and text analysis).

Chapter 4 consists of a report on TS1, which among other things includes answers to the two questions introducing this chapter. The first section presents our quantitative findings concerning the teachers' background (gender, age, education etc.) and their school mathematical priorities, i.e. what they value and think should be emphasised in school mathematics. In the second part of the chapter we analyse the teachers' response to the questionnaire qualitatively and present and discuss their comments on KappAbel in the open items of the questionnaire. In appendix A, Inge Henningen presents the statistical analyses of the data in TS1. In appendix B, you may find a technical report on data collection and handling in the teacher survey. You may find the questionnaire in appendix 1.1, the letter to the schools/headmasters in appendix 1.2, and the letter to the mathematics teachers in appendix 1.3.

4.1 The quantitative study

The teachers' background, experiences and participation in KappAbel

The survey (TS1) was conducted in December 2004 in between the two qualifying rounds of KappAbel. 944 mathematics teachers from all over the country participated in the survey, among these were 393 women (42%) and 549 men (58%), see table 4.1. Approximately 60% of the participating teachers worked at a school in a rural area, and around 40% in an urban area. A little more than half of the teachers were 45 years or more. Almost one third of the teachers were 55 years or more, while almost one fourth were younger than 35 years, and the rest of the teachers were equally distributed between 35 and 54 years. Approximately 40% of the teachers had less than 10 years of teaching experience in mathematics and around 20% had more than 30 years experience. The average number of teaching experience year was 16. A little more than half of the teachers are educated at the teacher training college, one third have a bachelor's degree, while the rest have another education e.g. master's degree. 10% of the teachers had no mathematics education at tertiary level, while 15% of the teachers had 5 Credits of mathematics (1/4 year). 34% had 10-15 Credits, while 41% had 20 Credits or more in mathematics.

In Year 9 of the Norwegian school, there are three mathematics lessons every week. In the academic year 2004-2005, 25% of the teachers responding the questionnaire taught 0-3 lessons of mathematics every week, 40% taught 4-6 lessons, 23% taught 7-9 lessons, and 12% of the teachers taught 10 lessons or more. 64% of teachers had a single class of Year 9 in mathematics, while 31% had two classes and the last 5% of the teachers had more.

The demographic teacher variables in table 4.1 (gender, age, education) have been compared, but no association between them has been found. For instance variables like age and academic background in mathematics are not associated; neither are gender and the number of mathematics lessons taught weekly. In this context it should be noticed that in the initial analysis gender were included as a possible confounding variable in all comparisons. In just a few instances we found significant differences between men's and women's response. In what follows we have marked explicitly if there is any difference related to gender.

In question 8, we asked the teachers how they perceive of themselves - as mathematicians, as teachers and/or as mathematics teachers. A little less than half (47%) ticked off disagree or neutral (-2 til 0) at the statement ”I am a mathematician”. In contrast to this almost everybody declared that they agreed with the statement ”I am a teacher” and like that – but less vigourously – in the statement ”I am a mathematics teacher”, see table 4.2.

Table 4.1 Teacher background and experience (number and per cent).

A. Background and experience

1. Gender: **female**: 393, 42% **male**: 549, 58%
   Number of answers in total 942

2. Year of birth: **-50:** 302, 32% **51-60**: 229, 24% **61-70**: 194, 20% **70-** : 219, 23%
   Number of answers in total 944

3.1 Education:
teacher training college: 509, 54% **bachelor**: 305, 53% **other education**: 119, 13%
Number of answers in total 933

3.2 The college level maths component of your education corresponds to approximately:

None	93	10%
1/4 year (5 credits)	47	15%
10-15 credits	322	34%
1 year (20 credits)	319	34%
20- credits	63	7%

Number of answers in total 925

4. I have taught/been teaching maths for _____ years.

0-4 years	205	22%
5-9 years	182	20%
10-19 years	149	16%
20-29 years	189	20%
30- years	206	22%

Number of answers in total 931

5. What other subjects do you teach? *We have not handled these data.*

6.1 This school year I am teaching ___________ weekly lessons of maths.

0-3 lessons	238	25%
4-6 lessons	371	40%
7-9 lessons	213	23%
10- lessons	111	12%

Number of answers in total 933

6.2 How many Year 9 mathematics classes are you teaching this school year?

0 classes	13	1%
1 classe	600	64%
2 classes	286	31%
3 classes	24	3%
4 classes	8	1%

Number of answers in total 931

7. My school is located in **an urban area**: 392, 42% **a rural area**: 546, 58%
   Number of answers in total 938

When the teachers have the possibility like this to tick off in a questionnaire, they first of all perceive of them selves as a teacher and next as a mathematics teacher and to a lesser extent mathematicians. In and by itself it is of interest that the teachers to a lesser extent identify with being mathematicians than with being teachers and teachers of mathematics. In our context, however, it is of interest to know whether KappAbel participation is associated with extent to which the teachers consider the subject of mathematics a significant determiner of their professional identity.

Table 4.2 The teachers answer to question 8 in TS1 (per cent).

How do you perceive yourself?	Disagree – Agree (per cent)					
	-2	**-1**	**0**	**1**	**2**	Answers
8.1 I am a mathematician	8	11	28	37	16	938
8.2 I am a teacher	0	1	1	17	81	909
8.3 I am a mathematics teacher	0	0	3	27	70	929

Who participate in KappAbel?

The teachers were asked the question of ”Are you teaching one or more classes that participate in KappAbel this year?”. 334 teachers (36%) said yes, and 604 (64%) no. Among the teachers who did participate, 65% had one class in the competition and 30% had two classes. In this context to participate in KappAbel means that the class has submitted an answer to the questions of the first qualifying round. According to the KappAbel statistics there was a loss of approximately one fourth of the classes between the two qualifying rounds in 2004/05 (see appendix 1.4).

We have looked at participation rate in KappAbel and compared it to the teacher variables (gender, age, education, credits and urban/rural area) (See table A.2 in appendix A). When the associations were controlled for the influence of other variables it was found that Credits in mathematics were the only variable showing a significant association with participation in KappAbel, meaning that higher credits in mathematics is associated with a higher participation rate (from 24% to 41%). (See table A.3 in appendix A.) This means that neither the teachers' gender, age, or education nor the location of the school have significant influence on their disposition to participate in the competition.

We asked the teachers whose classes did participate in KappAbel if their classes will be doing a project on Mathematics and the Human Body, even if they fail to make it to the semi-final of KappAbel. 22% responded yes and 78% no. 517 of the teachers (59%) said yes to question 17 on whether any of their previous classes participated in KappAbel. Among these teachers, 18 pct points participate in KappAbel 2004/05, while 41 pct points do not participate. 357 teachers (41%) answered that they do not have any previous experience with KappAbel. Among these 19 pct points participate in 2004/05, while the remaining 22 pct points do not participate this time either. This means that 19% of all the teachers in the study participate in KappAbel for the first time in 2004/5, while 22% have never participated in the competition (see table 4.3). Among the teachers whose classes did participate in KappAbel before, two thirds state that they have been doing project work on the KappAbel theme.

Among the teachers responding to this question, the disposition to participate is smaller than among those who have participated before (18/59) than among those who did not participate before (19/41).

Table 4.3 Teacher participation in KappAbel, past and present (per cent)

Did participate in KA **Participate in KA 04/05 **	**Yes**	**No**	**In total**
Yes	18	19	37
No	41	22	63
In total	59	41	100

Why participate/not participate in KappAbel?
334 of the teachers had classes participating in KappAbel. These teachers were asked to indicate their level of agreement with a range of possible reasons to participate (see table 4.4). 60 of the teachers wrote supplementary comments with reasons, explanations or considerations (see chapter 4.2).

Almost all teachers agreed that they hac their classes participate in order to motivate the students for mathematics. Approximately 90% agreed that the problems of KappAbel are challenging and considered this a reason for their particiation. Also approximately 90% participate because the competition offers opportunities for collaboration. Three fourths of the participating teachers agreed that the competition format offers an opportunity for some students to take a more active part in mathematics education. About 40% states they participate because their school has a tradition of participating in KappAbel. 25% strongly disagrees with this reason and this might be interpreted like this. One fourth of the teachers participating in 2004/05 is the only participant at their school.

Roughly 60% of the teachers disagree or are undecided on participating because the students wanted to participate. This answer could be compared with the answer to the corresponding item in question 15, which the teachers not participating in KappAbel have given. 80% disagree that they don't participate because the students do not want to participate. Together the answers suggest the students only to a little extent are involved in the decision on participating or not.

Table 4.4 Teachers' reasons for participating in KappAbel 2004/05 (per cent)

My class(es) is(are) participating in KappAbel		**Disagree – Agree (per cent)**					
		-2	**-1**	**0**	**1**	**2**	Answer
14.1	in order to stimulate student interest in mathematics	0	1	4	31	64	336
14.2	because the students wanted to participate	12	10	36	25	17	335
14.3	because the problems represent a challenge	0	0	9	46	45	332
14.4	because my school has a tradition of participating	25	8	26	25	16	332
14.5	because the competition offers opportunities for collaboration	1	3	8	43	45	335
14.6	because this year's topic on Mathematics and the Human Body is a source of inspiration for engaging in mathematical activities	18	9	49	7	7	336
14.7	because the competition format may offer an opportunity for some students to take a more active part	2	4	20	44	30	335

There are no top scores among the reasons for not participating formulated in the questionnaire. A little less than half of the teachers agree that a reason not to participate is that the project is too time-consuming in relation to what the students get out of it. A similar propotion agreed that there is no tradition for participation at their school. On the other hand almost all teachers disagreed or were undecided in that competitions generally are a bad idea (55% strongly disagreed), and similarly almost all teachers disagreed in that they do not participate because the problems are not good enough (46% strongly disagree).

Table 4.5 Teachers' reason for not participating in KappAbel 2004/05 (per cent)

My class(es) is(are) not participating in KappAbel		Disagree – Agree (per cent)					
		-2	**-1**	**0**	**1**	**2**	Answer
15.1	because competitions are generally a bad idea	55	20	23	1	1	575
15.2	because the students do not want to participate	27	13	40	12	8	574
15.3	because the problems are not good enough	46	15	38	1	0	572
15.4	because my school does not have a tradition of participating	19	8	27	24	22	588
15.5	because the project is too time-consuming in relation to what the students get out of it	15	10	29	30	16	582
15.6	because this year's topic of Mathematics and the Human Body does not represent a source of inspiration for engaging in mathematical activities	36	16	45	2	1	572
15.7	because the competition format may serve to isolate some students even further	24	13	40	18	15	572

In general non-participation does not primarily seem to be based on a rejection of the character of the competition and the tasks themselves. Also, the majority of the teachers (between 56% and 78%) find the three basic principles of the competition valuable while only 10% do not (see chapter 4.2 for a more detailed answer). However, many teachers do consider it a major problem that participation takes too much time in comparison with the potentials for students' learning.

Table 4.6 Teachers' attitudes to the three basic ideas in KappAbel (per cent)

It is a positive aspect of KappAbel that		Disagree – Agree (per cent)					
		-2	**-1**	**0**	**1**	**2**	Answer
16.1	the entire class collaborates and submits a common solution	3	3	16	27	51	868
16.2	the class undertakes a project based on a given topic	4	4	35	34	23	869
16.3	there are two boys and two girls on each finalist team	5	5	34	22	34	868

Table 4.7 Men's and women's attitudes to the three basic ideas in KappAbel (per cent)

It is a positive aspect of KappAbel that		Disagree – Agree (per cent)					
		-2 or -1		0		1 or 2	
		m	**w**	**m**	**w**	**m**	**W**
16.1	the entire class collaborates and submits a common solution	8	4	20	11	72	85
16.2	the class undertakes a project based on a given topic	9	6	37	32	54	62
16.3	there are two boys and two girls on each finalist team	11	9	40	26	49	65

The female teachers are in general more positive than the male teachers to the three basic ideas in KappAbel. There is a strong association with gender in all three items. Especially the principle of collaboration with a common solution and the principle of two boys and two girls on the finalist team from each fylke (Chi-square <.0001). There are 42% of the women who strongly agree (2) with equal sex distribution against 28% of the men (se Table 4.7). The other variables (age, education, credits, urban/rural) do not show any appreciable differences.

The teachers' school mathematical priorities

Questions 9 and 10 are about mathematics teaching and learning. They contain a series of statements on what mathematics in schools should put *heavy* emphasis on (question 9) and on possible characteristics of particularly *good* mathematics teaching that (question 10). Question 9 primarily concerns what the teacher regards as the main objective of the students' learning processes (see Table 4.8), whereas question 10 concerns the character of the instructional activities (see Table 4.9). Some of these statements fit particularly in with the reform discourse (see chapter 1.5). For example ”giving students the opportunity to discover and generalise about mathematical connections, rules and patterns” (item 9.2), ” letting students experience how mathematics can help them solve practical everyday problems” (item 9.6), “the students discover the world of mathematics together” (item 10.3); and ” the students explore and discuss before the teacher goes through the topic on the blackboard” (item 10.5). Between 62% and 85% of the participating teachers agree or strongly agree with theses four statements, whereas only 5% to 13% disagree.

At the same time almost all the teachers agree that mathematics in schools should put heavy emphasis on letting students practise the basic skills of the subject (item 9.11), and around three fourths agree that it is a characteristic of particularly good mathematics teaching that the students achieve routine proficiency in solving mathematical exercises (item 10.4), and that the teacher spends a good deal of time thoroughly explaining to the students the methods and concepts of the subject (item 10.1). Letting students learn mathematical formulas and methods applicable to real world problems (item 9.3) should be given high priority according to 90% of the teachers.

Table 4.8 Teachers' priorities in school mathematics (per cent)

Mathematics in schools should put *heavy* emphasis on		Disagree – Agree (per cent)					
		-2	**-1**	**0**	**1**	**2**	Answer
9.1	letting students learn mathematics relevant to their future careers	1	4	17	41	37	936
9.2	giving students the opportunity to discover and generalise about mathematical connections, rules and patterns	1	9	19	47	24	937
9.3	letting students learn mathematical formulas and methods applicable to real problems	0	2	8	44	46	937
9.4	letting students experiment in order to find and describe solutions to mathematical problems	2	5	18	49	26	938
9.5	letting students learn some of the classic mathematical proofs	5	13	26	42	14	935
9.6	letting students experience how mathematics can help them solve practical everyday problems	1	4	10	31	54	939
9.7	encouraging students to ask innovative mathematical questions	2	6	22	43	28	934
9.8	presenting mathematics to the students as logical connections and structures	1	3	11	47	38	935
9.9	letting students explore hypotheses on mathematical problems – their own as well as those of others	6	14	36	34	10	937
9.10	letting students work on logical mathematical reasoning	0	4	15	52	29	939
9.11	letting students practise the basic skills of the subject	0	1	2	24	73	939

Table 4.9 Teachers' attitudes to characteristics of good mathematics teaching (per cent)

It is a characteristic trait of particularly *good* mathematics teaching that		Disagree – Agree (per cent)					
		-2	**-1**	**0**	**1**	**2**	Answer
10.1	the teacher spends a good deal of time on thoroughly explaining to the students the methods and concepts of the subject	2	7	17	43	31	936
10.2	the students discuss each others' suggestions and methods	1	5	20	49	25	936
10.3	the students discover the world of mathematics together	3	7	28	41	21	932
10.4	the students achieve routine proficiency in solving mathematical exercises	1	6	15	43	35	937
10.5	the students explore and discuss before the teacher goes through the topic on the blackboard	4	9	29	41	17	935
10.6	the students learn to co-operate in solving mathematical problems	0	2	14	49	35	931
10.7	the students work, among other things, on projects that include subjects in addition to mathematics	4	7	23	43	23	932

Factor analysis of the teachers' school mathematical priorities

The teachers' responses to questions 9 and 10 are interrelated and for each of the questions we used factor analysis as a way of reducing the dimensionality of the information and finding latent structures in the data (see Appendix A).

Mathematics in school should put heavy emphasis on

The analysis of the 11 items in question 9 with three factors were performed and gave the factor pattern in table 4.10. Values over 45 are highlighted to indicate that the item gives a high contribution to the factor in question.

Tabel 4.10 Factor scores for statements on what mathematics in schools should put *heavy* emphasis on

Item (Statement)		Score		
		Factor 1	Factor 2	Factor 3
9.2	giving students the opportunity to discover and generalize about mathematical connections, rules and patterns	**66**	10	20
9.4	letting students experiment in order to find and describe solutions to mathematical problems	**64**	12	21
9.9	letting students explore hypotheses on mathematical problems – their own as well as those of others	**60**	44	14
9.7	encouraging students to ask innovative mathematical questions	**49**	38	16
9.10	letting students work on logical mathematical reasoning	24	**62**	14
9.8	presenting mathematics to the students as logical connections and structures	11	**58**	16
9.6	letting students experience how mathematics can help them solve practical everyday problems	31	21	**49**
9.3	letting students learn mathematical formulas and methods applicable to real problems	13	25	**47**
9.1	letting students learn mathematics relevant to their future careers.	15	4	**46**
9.11	letting students practise the basic skills of the subject	3	35	34
9.5	letting students learn some of the classic mathematical proofs	23	29	13

Table 4.11 summarises the results from table 4.10 giving the main content of each of the three factors. The first factor is seen to represent attitudes connected with the discovery of rules and patterns, with investigations and formulation of new mathematical questions and is termed *patterns and investigations*. The second factor is primarily connected with logic and structure and is termed *structures and logical reasoning.* The third factor is connected with practical

everyday problems, formulas and methods and relevance to future work, hence it represents a *tools and applications* dimension. Almost all teachers (97%) find item 9.11 ”letting students practice the basic skills of the subject” important. Hence this item does not correlate specifically with any of the three dimensions. Neither do the item 9.5 “letting students learn some of the classic mathematical proofs” but this is not because all teachers find it important.

Table 4.11 Factor content: The three value dimensions constructed in the factor analysis of TS1 question 9

Dimension	**Name/value**	**Key items**
Factor 1	Patterns and investigations	Discover rules and patterns, experiment, explore, ask
Factor 2	Structures and logical reasoning	Logical reasoning, logical connections and structures
Factor 3	Tools and applications	Practical everyday problems, formula and methods, relevant to future work

Considering the character of the tripartite scheme that led to the formulation of the questionnaire (cf. chapter 3), it is hardly surprising that the factor analysis led to these three factors: They are clearly associated with the distinctions between mathematics as a process, a system, and a toolbox. Our main interest, however, was not to confirm or otherwise the relevance of these distinctions. Rather it was whether teachers who claim agreement with one of these alternatives when presented with them in a questionnaire are more likely than their colleagues to participate in KappAbel

The differences between the teachers who did or did not participate in the KappAbel competition were analysed for each factor (dimension) (see appendix A). We found a small but statistically significant difference for the dimension designating patterns and investigations, in the sense that teachers participating in KappAbel have a slightly higher score on this dimension. For the two other dimensions there is no association between score and participation in KappAbel. The quantitative study, then, does not suggest that KappAbel is confirmative rather than formative, in the sense that teachers who comply with the rhetoric of the reform to a greater extent than their colleagues participate in the competition.

In other terms, participation appears to be based on issues other than those related to their views of mathematics, at least as conceived in the questionnaire.

Characteristics of particularly good mathematics teaching
A factor analysis of the seven items in question 10 with two factors was performed and gave the factor pattern in table 4.12 (see appendix A). Values over 45 are highlighted to indicate that the item gives a high contribution to the factor in question.

Table 4.12 Factor scores for statements on characteristic traits of particularly *good* mathematics teaching that (question 10 in TS1)

Item (Statements)		Score	
		Factor 1	Factor 2
10.2	the students discuss each others' suggestions and methods	**74**	-1
10.3	the students discover the world of mathematics together	**68**	-4
10.6	the students learn to co-operate in solving mathematical problems	**67**	2
10.5	the students explore and discuss before the teacher goes through the topic on the blackboard	**63**	-2
10.7	the students work, among other things, on projects that include subjects in addition to mathematics	**45**	-10
10.1	the teacher spends a good deal of time on thoroughly explaining to the students the methods and concepts of the subject	-10	**54**
10.4	the students achieve routine proficiency in solving mathematical exercises	4	**53**

Table 4.12 summarises the results from table 4.11 giving the main content of each of the two factors. The first factor is seen to represent attitudes connected with discussion and co-operation between students, is termed a *collaboration* dimension. The second factor is primarily connected with the teacher explaining

the methods and concepts of the subject and the students achieving routine proficiency in solving mathematical exercises. It is termed a *presentation and practice* dimension.

The differences between the teachers that did or did not participate in the KappAbel competition were analysed for each factor (dimension) but we did not find any significant association between score and participation for any of the dimensions. As above, there is no indication that KappAbel is confirmative rather than formative.

Table 4.13 Factor content: The two value dimensions constructed in the factor analysis of TS1 question 10

Dimension	**Name/value**	**Key items**
Factor 1	Collaboration	Discussion, co-operation
Factor 2	Presentation and Practice	Explain methods, routine proficiency

This reduction of dimensions of question 9 and question 20 in the TS1 questionnaire was used to make a qualified reduction of the number of items in the corresponding questions on learning mathematics (9) and teaching mathematics (10) in the TS2 questionnaire. The number of items is reduced from 11 to four and from seven to three respectively.

Representativity

The population for Teacher Survey 1 (TS1) was all teachers teaching grade 9 mathematics in the academic year 2004/05. No central registration of the teachers existed. Hence it was not possible to obtain a list of names and addresses of the teachers. Moreover, the size of the population was not known. There is no central registration or statistics of this particular group of teachers. The size and the composition of the population (gender, age, education etc.) and the geographical distribution (urban/rural and county) were not known, hence it was not possible to calculate the non-response rate directly. Nor was it possible to assess the representativity of the respondents comparing variables such as gender and school location with that in the total population.

However, in TIMSS 2003 (Grønmo et al., 2004) Norwegian mathematics teachers at grade 8 responded to the question whether they had “absorption” in mathematics (Credits more than or equal to 20). It turned out that 37% of the students in grade 8 were taught mathematics by teachers who had “absorption” in the subject. In TS1, 1276 classes at grade 9 are represented, among these 545 classes – corresponding to 43% – had teachers have “absorption” in mathematics.

According to Norsk Skoleinformation (2004) there were 2851 grade 9 classes[1] in Norway in 2004-2005. The questionnaire has been answered by 944 teachers, who between them taught 1276 classes corresponding to 44,8% of all classes. We presume that the teacher’s disposition to answer the questionnaire does not correlate with the number of classes he/she had in 2004/05. Thus we estimate the response rate for the teachers to be 45%.

The participation rate in KappAbel among the teachers who answered the questionnaire was 35,6%. This is somewhat higher than the participation rate among all grade 9 classes, where the participation rate was 25,3% (720 classes out of 2851, data from the KappAbel Secretariat, see appendix 1.4 and B).

The focus of the present quantitative study is the possible asssociations between demographic teacher variables and teachers’ educational priorities on the one hand and their participation in KappAbel on the other. In the analysis, we make the assumption that we do not have a differential non-response rate on the interaction level. This means that even if the response rate differs for instance between men and women and between participants and non-participants in KappAbel it might still be warranted to assume that the association between gender and participation will be the same for respondents and non-respondents (see appendix A).

Letters were sent to 1189 lower secondary schools in total. In each letter there were 1-7 questionnaires corresponding to the number of classes at grade 9. We received answer from teachers from 646 or 54% of these schools. We saw only moderate differences in participation rate between the 19 counties. The lowest participation rate was found in Finnmark (41) and the highest in Oppland (72). The 14 Southern counties have a participation rate between 56 and 72, except Aust-Agder (48%). Each school in the Southern part of Norway had on average 2-3 classes at grade 9 in the academic year 2005-2006, while the schools in the North had only 1-2 classes. Thus, there were more teachers per school in the population in the South than in the North and this might explain some of the difference.

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[1] A number reduced by us – see appendix B.

Our general estimation of representativity of the received answers in TS1 is that it might not be warranted to generalise toot strongly neither results concerning the demographic composition of the population, their motives for participating in KappAbel nor their attitudes to teaching and learning mathematics – despite the big number of respondents (944). On the other hand we expect that results concerning association between teacher variables and between teacher background and participation in Kappabel are generalizable to the total population.

4.2 Taking the qualitative remarks into account

253 of the teachers in TS 1 submitted qualitative remarks of some kind. These vary from a pleasant, but uninformative 'Merry Christmas' to 15 lines of significant comments (transcribed) on topics ranging from how the teacher in question sees KappAbel, over comments about his of her general view of the (lack of) the potential of project work and about the administrative problems of local schools or authorities, to explicit comments about the (lack of) relevance and significance of the research project. There are only three of the teachers who respond by claiming that in general they disagree with the overall priorities of the competition. The vast majority of those who contribute with qualitative comments do so by arguing either that they are pleased with some or all of the priorities of KappAbel and they are happy to have their classes participate or that they are unable to do so for some specific reason. In all of the comments, three main issues related to the research question emerged, namely the ones of a time factor, of the contents and the approach of KappAbel, and of lack of knowledge and information.

The time factor

70 of the respondents to TS1 wrote comments related to the factor of time (28% of the teachers with comments). The majority of these comments were concerned with a perceived pressure to cover the contents of the syllabus, a pressure that was acute because the expectations in LP97 (see chapter 1.1) appeared to be incompatible with only teaching three mathematics lessons a week in grade 9.

We barely have time to do what we have to do!
(Female in her late 40s with a college background with little mathematics)

The main problem is TIME!
(Male in his late 50s with a comprehensive mathematical background working in a town or city).

The amount of mathematics and the thematic structure of the syllabus leave us with little room to deviate from 'ordinary' teaching. There should be more opportunities of deviating [from the syllabus] and to become absorbed [in maths].
(Male in his early 60s, with relatively poor mathematical background)

I experience an ever increasing problem with mathematics lessons that vanish and are used for other imposed purposes. In grade 9 in our school there is in reality only 2 lessons a week for mathematics. I only have time for what is absolutely essential. UNFORTUNATELY!
(Female in her early 50s with a fairly comprehensive mathematics background)

The contents in grade 9 are extensive and the number of lessons, 3 a week, is way too small. That limits what we can do.
(Female in her early 60s with a college background with an intermediate level of mathematics)

Initially the class did take part, but when they were to submit the students decided to prioritise revision and work directed towards the autumn mock exam. Besides there are only 3 lessons a week in grade 9 mathematics -> somewhat hectic.
(Male in his early 30s with a university background with little mathematics. He works in a town or city.)

These teachers all share an experience that the combination of the present syllabus and the number of lessons available for mathematics in grade 9 does not leave room for involvement in KappAbel. A closer reading of the responses, however, indicate that the apparent unanimity among these teachers is to some extent just apparent and covers a wide range of different school mathematical priorities. There are, then, a number of different aspects to the issue of time. This becomes evident once the qualitative comments are linked to the categorisation of teachers that was based on the quantitative responses, cf. section 4.1.

Nina is a college graduate in her mid 20s, who is only just into her second year of teaching. She did the intermediate level of mathematics at college and strongly identifies with being a teacher of mathematics and with being a teacher in general. However, she distances herself from being considered a mathematician. Explaining why she decided not to let her two grade 9 mathematics classes take part in KappAbel, she says:

It is difficult to take your time to do competitions and bigger projects when you know in advance that you don't have time enough (3 lessons a week) to cover the curriculum.

Elaborating on the issue of time, she says that the amount of time spent on project work is not justified by the students' learning. Also, Nina says that there is no tradition of participation in KappAbel at her school (question 15), and that she disagrees with the feature of KappAbel that the class submits a joint solution to the project (question 16). Nina generally prioritises the tools and applications dimension of the subject, while the logical/structural and investigative sides are considered less important (question 9). As far as teaching and classroom culture is concerned, she considers discussions and co-operation important, while she considers teacher exposition and students' practice of routines relatively less significant, though still of some importance (question 10).

When seen from the perspective of Nina's school mathematical priorities as evidenced in TS1, an interpretation may be made of her reluctance to engage in KappAbel because of time constraints. It seems that it may be linked to her emphasis on certain aspects of her conception of (school) mathematics, and not to the type of interaction, she values in mathematics education. More specifically, it may be conjectured that her emphasis on mathematics for everyday life (incl. working life) and on the basic skills is seen as at odds with she considers the most significant aspects of KappAbel.

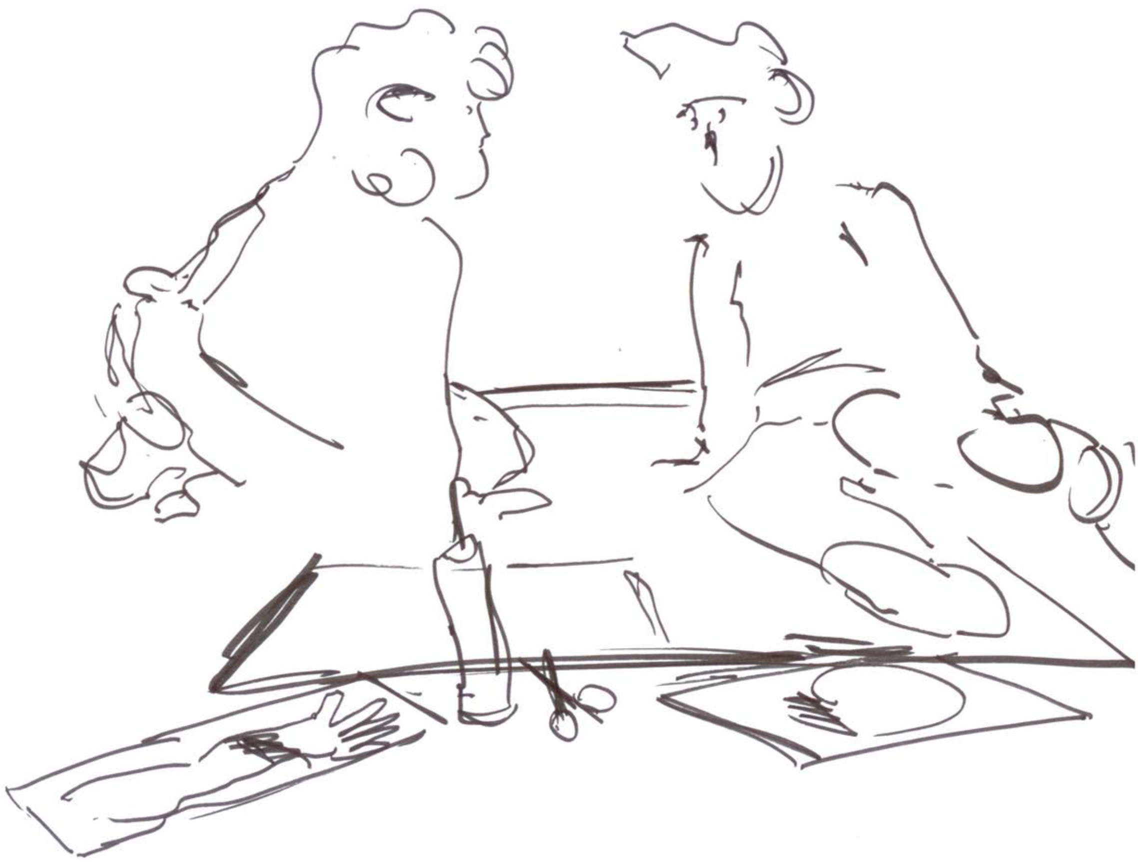
Karen is in her late 40s. She is a college graduate with intermediate mathematics background. She identifies herself with being a teacher and a teacher of mathematics, but she does not consider herself a mathematician. She decided not to let her grade 9 mathematics class take part in KappAbel. Explaining why, she said:

The reason for not participating in KappAbel is that we have been engaged in another project.

The mathematics curriculum in lower secondary school has become so comprehensive that we have speed up in order to make it.

Many students lack basic skills in the subject when beginning in lower secondary school. This has become a problem that is getting still worse.

Karen was one of the teachers who prioritise the dimension of structures and mathematical reasoning. She consistently prioritises the logic and structure of mathematics as well as the utilitaristic aspects of the subject. Further she deemphasises the more investigative sides to it (question 9). Also, she identifies good mathematics teaching with teacher exposition of the contents and explanations of the students' work. Surprisingly, she indicates to disagree strongly with all the reasons suggested for not participating (question 15), and agrees that the project element of KappAbel is a good idea (question 16). In spite of that she claims that the time available does not allow her to become involved in KappAbel. There is no indication of what type of project the Karen's grade 9 was involved in (cf. the quotation).



Karl is a 60 year old teacher with a university degree, but not in mathematics. He has intermediate level background in mathematics and considers himself a mathematician as well as a teacher and a teacher of mathematics (question 8). Explaining why he chose not take part in KappAbel with his two grade 9 mathematics classes he says:

> With only 3 lessons a week in grade, there would be too little time to take part in KappAbel.

Karl prioritises applications of mathematics. He gives high priority to the students' mastery of basic skills and their preparation for their future working lives (question 9). In terms of teaching, he gives high priority to teacher explication of concepts and procedures and to the students' subsequent practice of these (question 10). In line with this, Karl does not feel strongly about the three main emphases of KappAbel (the joint solution by the whole class; the thematic project work; gender co-operation), and he is convinced that the time spent when engaging in project work far exceeds the possible benefits in terms of student learning.

Karl's school mathematical priorities, then, are in all respects at odds with the KappAbel competition.

Both Nina and Karl claim that the lack of time is the main reason why they do not choose to engage in KappAbel together with their grade 9 students. However, the time factor is in these cases clearly related to a set of school mathematical priorities that seem to be at odds with those of KappAbel. For these teachers, then, time is not the only factor and probably not even the most important one. It seems that the time factor is used to legitimise a lack of involvement that is more closely linked to their educational priorities. KappAbel is for these teachers an additional task, something that may be included when the obligations related to what is conceived as the more significant aspects of school mathematics have been dealt with. KappAbel does for these teachers not appear to have the potential to become a significant contributor to the students' mathematical learning. For all its motivational benefits, it remains a treat and a luxury that is relevant only if the more serious challenges of school mathematics have been dealt with. This seems to be the case for a large proportion of the teachers. It may be conjectured that for them, a realistic reduction of the contents of the syllabus or an extension of the number of teaching hours is unlikely in and by itself to significantly increase their propensity to participate in activities like KappAbel: there is always a need for more time for practising the basic skills if that is the main priority.

Also emphasising the time factor, Sylvia, a teacher in her mid 50s with a college background with intermediate level mathematics says:

> I find it difficult to answer questions 15 and 16 [the ones suggesting reasons not to participate in KappAbel and describing level of (dis-)agreement with the main characteristics of KappAbel]. But it is difficult with only 3 mathematics lessons a week. On top of that, much is 'lost' for other purposes in the school of today, it just has to be like that [i.e. that they do not participate]. If we had many mathematics lessons, I would probably have judged this differently.

Sylvia, then, is explicit that his decision not to participate is immediately related to the limited number of lessons, and that he would probably have decided otherwise, had there been more. This may be seen as different from the situation with for instance Nina and Karl (cf. above). However it seems similar to that of Christina, a teacher in her early 40s, who also has a strong background in mathematics, She says:

> There is no room for KappAbel this year; many other projects/themes going on.

Like Nina, Christina prioritises the students' command over the basic skills and their acquisition of methods that are readily applicable to everyday life. However, she also presents an image of school mathematics that is significantly more investigative than Nina's: she consistently emphasises the investigative aspects of the school subject (question 9). However, her priorities as far as teaching is concerned does not in general emphasise student involvement in investigations and discussions, although she does disagree that the learning outcome of project work does not correspond well to the effort in terms of time. This is reiterated as she considers project work in KappAbel a good idea (question 16).

There are 17 teachers like Christina, who claim to opt out of KappAbel because they are involved in other projects and do not have time to do both. These teachers differ significantly in terms their other school mathematical priorities as depicted in TS1. Some of them appear to have very different priorities than KappAbel, somewhat in the same sense as Nina and Karl. Others, like Christina, also claim to be doing projects, and combine that with a much stronger process orientation of their mathematical or educational approach. In these cases one may speculate that there is a stronger potential influence of KappAbel, because of the immediate resonance of KappAbel's priorities with those of the teachers themselves. Although none of the teachers give any hints about the mathematical and educational priorities of these other projects, there is evidence of significant differences between the teachers who point to time as a limiting factor. In some cases they seem to comply with KappAbel and more time is likely to have an influence on their future participation, whereas in others this seems not to be the case.

One may argue that it is not a serious problem for KappAbel, if teachers opt out of the competition because of their involvement in other projects developed locally or regionally. The validity of that argument, of course, depends on the extent to which the priorities of these other projects correspond with those of KappAbel. It is beyond the present study to shed light on the extent to which that is the case. In general, however, there is a serious issue of time involved in participations in KappAbel. Also the teachers who do not mention their involvement in other projects claim that time is a serious limitation, and among all the teachers who write additional comments in TS1 time is by far the factor most frequently mentioned. Even a few of the teachers who consider themselves in line with the priorities of KappAbel and whose classes did well in the competition claim that their classes would not have taken part in the project on *Mathematics and the Human Body*, if they had not advanced to the national semi-finals.

The tasks, contents and approach of KappAbel

51 of the teachers commented on the character and quality of the tasks and problems presented in KappAbel and on the suitability of the approach adopted for the students in question. To some extent these issues are related to the one of time discussed above, but this is not the main emphasis. Rather the teachers discuss how their students react to the approach adopted, and how they themselves decide whether to become involved based on the students actual or expected reactions.

> I experience a growth of motivation in mathematics because of the KappAbel tasks. The students solve more tasks. I also think the classroom environment benefits. […] Finally: I thank you for a wonderful and tremendously valuable contribution to my teaching of mathematics.
> (Male in his mid 50s with a university background and intermediate level of mathematics)

> We don't participate because it is a very small class and among the students there are a couple with very poor qualifications in mathematics.
> (Male in his early 30s with a university degree with little mathematics)

> We use *the nut of week*, many find it exciting. The competition is discussed a long time after.
> (Female in her late 50s with a college background with a high level of mathematics contents)

> KappAbel is too difficult for most of the students (→ noise).
> (Female in her mid 30s with a college background and intermediate level of mathematics)

These comments suggest very different expectations to and experiences with KappAbel. A relatively modest number of teachers (<10) are explicit that participation in the competition changes their everyday teaching, while a considerably higher number of teachers (25-30) suggest that the use of *the nut of the week* or other problems from KappAbel motivates their students and boosts their own teaching by adding a longed for occasional variation to the way of working. This is so for instance for Monica. She is a teacher in her early 30s with a college background and intermediate level content preparation. Monica’s grade 9 classes participated in KappAbel, because she considers it motivating and challenging for the students and invites types of co-operation that allow students, who do not normally play a strong role in mathematics the opportunity to do so (question 14). Also, she agrees with the cooperative spirit reflected in the requirement to submit a joint solution and especially with the idea that the teams for the finals are to consist of two boys and two girls. In Monica’s own wording KappAbel consists of

> [R]eally good and engaging tasks. Different from the usual ones. We also use the nut of the week on the web.

Jon is a college graduate of about 30. He has a fairly strong mathematical background. Like Monica, he uses the tasks developed for KappAbel:

> [I]collect tasks from Kappabel, but do not participate

In spite of his use of the tasks, then, Jon has decided that his students should not participate in the competition. He claims that there is no tradition of such participation at his school, and he disagrees with the ideas of the class submitting a joint solution and of basing part of the competition on a larger project. Monica and Jon may be seen as examples of teachers who allow for different potential influences of KappAbel on mathematics teaching and learning. Monica engages in KappAbel as a whole, Jon uses only specific tasks to further develop his teaching within a more traditional overall framework. Both of them, however, seem to see KappAbel as an important source of inspiration.

This positive evaluation of KappAbel and its potential influence on the classroom practices are challenged by other teachers who claim that KappAbel is not a competition for the average student in grade 9 and especially not for their own students:

> We have down to 5 students in each class; it then becomes unfair in comparison with bigger classes. We have too few players in comparison with classes of up to 30 students.

> At our school there are only 5 girls in grade 9. That makes it difficult to fulfil the requirements [i.e. that there needs to be two girls and two boys in each of the groups].

In these cases explicit reference is made to the small classes of quite a few countryside schools north of More and Romsdal. The teachers argue that due to the small size of the classes, some of the organisational requirements of KappAbel do not fit the local context. These are the requirements of the whole class working together and of the class being represented by two boys and two girls. The two teachers quoted, then, do not explicitly disagree with these requirements in general, but base their rejection of the organisational requirement on the situation at their own school.

Other teachers refer to the problems related to the contents of KappAbel, or rather to problems related to the relationship between the contents and the students they teach:

> KappAbel is a competition for students who are interested in mathematics. We have many such students. But we also have a big group of students who are not particularly engaged in the subject. – maybe they struggle with mathematics. For these students, KappAbel will become an activity with something that they would prefer not to do.

> At our school we have many students with big and complex problems. Many of them have holes* in their mathematical skills and the level in KappAbel is too high. Most of our students lack the very basic skills.

There are 26 teachers who argue that in general the KappAbel competition is too difficult. However a significant number argue like the ones above that KappAbel is particular inappropriate for the students at the particular school. This may be seen as an acknowledgement that schools and students are different, and that as a teacher you should only speak with confidence about the situation you know the best. However, one may also speculate that the teachers' response involves an element of legitimising their own lack of participation, at least as far as the problem of the level of difficulty of the tasks is concerned. If so, they do it by making a local argument that does not involve them in considerations about the general (lack of) qualities of KappAbel[2].

It is beyond the design of the present study to shed light on the extent to which this latter concern is valid. In any case, it is notable that a significant number of teachers claim that KappAbel is not suited to the students of grade 9 as they know them. It is unfortunate for any educational initiative, if a sizeable

[2] To the extent this is the case, the questionnaire may have had an element of symbolic violence, as the use of it confronted the teachers with a different set of priorities and imposed a situation in which they were in effect required to legitimise their current instructional choices.

proportion of the teachers declare it unfit for the students for whom it is intended. In the case of KappAbel this is even so, as the teachers' comments that the tasks and organisation of KappAbel are too difficult to be relevant for their students seriously challenge the extent to which the competition manages to fulfil its promises of being inclusive. These are comments that seriously challenge the self-conception of the competition.

Lack of knowledge and information

One further set of issues were raised by a number of respondents to TS1: they claimed to have insufficient knowledge or information about KappAbel or to have received information too late for their classes to participate. As is the case with the other sets of comments, the headline of lack of information conceals what appears to be a range of different reasons for not participating in the competition.

> […] this is the first school year in which we teach grade 9, so I haven't familiarised myself with KappAbel. We shall probably join later (grade 9 is only 6 students this year). [I] Find KappAbel good!
> (female in her early 30s, with a college background and intermediate level mathematics).

> Very often we […] experience that information about projects like these never reach the teachers. At our school, the reason is the municipal administration and the headmaster. [We did not participate] BECAUSE THE SCHOOL HAS NOT INFORMED US ABOUT KAPP-ABEL! INFORMATION BREAKDOWN (NOT UNCOMMON IN NORWEGIAN SCHOOLS).
> (Male in his early 60s with a university background and a high level of mathematics)

> Know too little about KappAbel..
> (Female around 30 with a college background and intermediate level mathematics)

There are altogether 23 teachers who claim that lack of information or insufficient knowledge is the main reason for them not to participate. Like Helene, the first respondent quoted above, some of these seem to potentially interested and have not acquainted themselves with the competition for fairly obvious reasons, in Helene's case that the school has never had a grade 9 before. Lack of participation is in the case of this teacher also connected to some of the organisational issues mentioned in the previous subsection, namely the one of having very few students in grade 9.

For Kristian, the second of the three teachers quoted above, the situation is different. He is not familiar with KappAbel and blames the local management for his ignorance. It should be noted that Kristian's priorities - as read off his responses to other questions in the questionnaire - do not at all seem to be in line those of KappAbel. For instance, he does not prioritise any of the process oriented items in question 9, while he appears to orient himself strongly towards both the logical relationships and the applications of mathematics. In question 10, he agrees that to some extent the students should cooperate when doing mathematics, but gives higher priority to teacher explanations and routines than to students' investigations and discussions and to project work. Apart from the quotation above, he gave no reasons why his classes did not participate.

Johan, a young teacher of around 30, represents a third group of teachers, who refer to lack of information at the right point in time as the main reason for their classes not to participate. He said:

> The class and the team [of teachers] has already made a plan for the year, which makes it awkward to make KappAbel's project work fit in the plan! If we have information of the theme of the KappAbel project work earlier (May/June, not September/October) it is easier to make it fit in the plan for the year.
> (Male of around 30 with a college background with a high level of mathematics contents)

Like Kristian, however, Johan also sees himself in opposition to the investigative and process oriented emphases in KappAbel. He consistently disagrees or disagrees strongly with the process items in question 9. But he also disagrees with those items referring to the more structural aspects of the subject, while giving high priority to the applicational ones. These responses appear to be easily interpretable from the point of view adopted in the study: Johan signals a strongly utilitaristic view of mathematics. In question 10 he agrees only that good teaching is especially characterised by students acquiring routine in solving tasks and that they should do projects that include connections to problems beyond mathematics itself. To the extent that he sees 'projects' as instructional sequences that will be specifically directed towards using mathematics in real world contexts, this appears to be in line with his utilitaristic priorities in question 9. Somewhat in contradiction to this response to question 10, Johan also claims that one of the reasons for his classes not to participate is that project work is not an efficient use of time. The only other reason is that there is no tradition at his school for participating.

Another teacher, Ole is in his 60s and has taught mathematics for almost 40 years. He also mentions his lack of knowledge of KappAbel, but does so in different terms from the ones above. He says:

> I consistently discard all mail except reasonably intelligent questionnaires. That's why I don't know anything about KappAbel. I shall never find out either.
>
> We need every precious moment to calculate our way through mathematics.

In his response to the question of why his classes do not participate, Ole agrees with the two items stating that the tasks in KappAbel are poor and that the time required for project work is excessive in comparison with the benefits in terms of student learning. Ole's chosen ignorance of KappAbel, then, goes hand in hand with his lack of identification with the priorities of the competition. However, Ole also makes another comment to the questionnaire that may shed further light on his lack of participation. He adds an item to the list of possible characteristics of good mathematics teaching in question 10: the classroom should "be completely quiet at all times". This may be read as though the way Ole distances himself from the mathematical priorities of KappAbel is supplemented by his rejection of the type of teaching-learning processes that he seems to associate with for instance project work, in which students are expected to some extent to organise their individual and collective mental and physical activity. In other terms, the last comment quoted indicates that his rejection of participating in the competition is also based on and profoundly influenced by his view of what in general constitutes a suitable classroom environment and an appropriate learning atmosphere.

Although Ole's statements are extreme in comparison to those of all other respondents to TS1, one may speculate that one of his comments may be indicative of a more general set of worries. His point appears to be that the classroom atmosphere encouraged and sustained by initiatives like KappAbel is incompatible with his vision of a supportive classroom environment. From TS1 it is impossible to judge whether this is an expression of an approach to education that deems subject matter learning as less important than for instance learning to behave, or whether Ole's comments are based on a vision of mathematics classrooms as settings for quiet contemplation and practice of the concepts and procedures of mathematics. In any case, it may be seen as an indication that his school mathematical priorities are not the sole determinants of his evaluation of KappAbel.

Other comments related to participation or non-participation in KappAbel

We have presented three sets of qualitative comments that were common enough to warrant a grouping of the respondents. These are the ones of lack of time in view of other obligations in grade 9 mathematics; of the quality and level of difficulty of the KappAbel tasks in relation to the students in question; and the availability of information and knowledge about KappAbel and what it has to offer.

A number of respondents gave other individual comments, which in different ways relate to the possible impact of KappAbel in schools. We shall mention a few of these.

Bjørn is a university graduate in his early 60s with a comprehensive background in mathematics. He states that more time and continuity is needed in mathematics teaching and that “exact knowledge” should be emphasised in order to make the students “happy with numbers”. Although Bjørn relates to the issue of time, his comments are explicit that the priorities of KappAbel are different from his own. The main issue, then, is not one of time, but of the profile of KappAbel being seen as counterproductive to what should be the main emphasis in school mathematics. It is in line with this interpretation that Bjørn only prioritises only three of the items in question 9, namely the two of the ones on structure and the one of the students practising the basic skills. In question 10, he agrees or agrees completely with all items, except 10.7 which is on the need for the students to become involved in project work beyond mathematics itself. The only item in question 10 with which he agrees completely is 10.4 on the need for the students to get routine in solving tasks.

Susan is a very experienced teacher in her early 50s. Like many of the others, she relates to time, but in a very different sense than the others. She says:

> The reason that I don’t consider to taking part is that I work about 60 hours/week on average. This [participation in KappAbel] would be on top of that.

Susan’s responses to questions 9 and 10 are not easily interpretable from the point of view of the study. However it may be indicative that she disagrees with only one item in question 9, the one on the need for the students to “investigate their own and their friends’ conjectures about mathematical questions“ (9.9). In question 10, she prioritises teacher explanations (10.1) and students’ routine work (10.4). Somewhat in opposition to the general picture that emerges, the only other item with which she claims to agree is that “students should investigate and discuss before the teacher presents the content”. If one reads this last response to have special emphasis on the latter half of the wording of the item, it may support an overall interpretation of a fairly traditional set of priorities in relation to teaching-learning practices. In line with this interpretation, one may speculate that she does not see KappAbel as an initiative that is in line with her own educational priorities and that therefore it does become an extra burden on her. Otherwise it may be that she is unfamiliar with the basics of KappAbel and that the basic hurdle of becoming acquainted with it is beyond her, when she sees herself caught up in so many other obligations. In any case, Susan appears to see herself as a highly committed teacher, who works too hard already. It is notable that she does not see KappAbel as an initiative that may support her and take off some of the pressure that she experiences by providing valuable input to her teaching; input that she is otherwise obliged to provide herself. Put shortly, KappAbel is not seen as a support system for her to make it easier for her to conduct quality teaching; rather it is an extra obligation that puts extra pressure not on her students, but on herself.

Torkel is in his mid 50s. He graduated from university with an intermediate level of mathematics. He has now taught the subject for more than 20 years. His attitude to KappAbel appears in strong opposition to Susan's as quoted above His final comment in the questionnaire reads:

> Thank you for a super offer and a valuable contribution to my maths teaching! My experience is that the students' motivation bloom when we use the KappAbel tasks. They solve more tasks, and the atmosphere in the class is improved. For the students' motivation I think it would be valuable, if the results of the first round had been put on the internet.

Torkel's evaluation of the role of KappAbel for his teaching may be surprising if one considers his responses to some of the quantitative items. He prioritises structures and applications and for instance agrees with all items in question 9 except two of the ones pointing to the investigative aspects of the subject, namely the ones on the students having "opportunity to find and generalise connections, rules and patterns" (9.2) and investigating "their own and their friends' conjectures about mathematical questions" (9.9). In question 10 he prioritises only the ones on the teacher's explanations (10.1); students' acquisition of routine in solving tasks (10.4), and students investigating and discussing before the teacher presents the content.

Conclusions

In TS1 we found a number of responses that revolved around three themes. These are the ones of

- lack of time in view of other obligations in grade 9 mathematics;
- the quality and level of difficulty of the KappAbel tasks in relation to the students in question; and
- the availability of information and knowledge about KappAbel and what it has to offer.
- 150 respondents, app. 60% of those who wrote qualitative comments or 15% of all those who responded to TS1, could be grouped in one of these three categories.

Altogether 28% of the teachers responding qualitatively claim that lack of time is a considerable problem for their participation in KappAbel. These teachers in general argue that the requirements of grade 9 mathematics are out of proportion to only having three lessons a week. A minority of these teachers indicate agreement with what we interpret as the educational priorities of KappAbel in the quantitative items of the questionnaire, in particular the investigative aspects or collaborative efforts. For these teachers it is reasonable to assume that time is perceived as a very real constraint in the sense that at present it is impossible to participate in KappAbel, but if a modest increase in time becomes available they would reconsider and possibly opt for KappAbel.

However, the majority of the teachers, who claim that time is a problem, do not – in our interpretation - see themselves as in line with the priorities of KappAbel. For these teachers, neither the mathematical emphases nor the collaborative concerns of KappAbel appear to be on the agenda. Consequently one may speculate that even considerable increases in time would not allow for comprehensive involvement in activities like KappAbel[3].

For both these subgroups it may be argued that the most ambitious, realistic expectation on the part of KappAbel is to inspire teachers to use a problem solving task now and then. This means that "the nut of the week" or similar ways of supplementing more traditional teaching-learning processes may be the most significant potential of KappAbel. One may speculate that this supplements dominant individual and collective activities and modes of interaction and therefore indicates ways of being in mathematics classrooms that challenge present classroom practices. There may, then, be an impact of KappAbel on these classrooms. However, the odd problem used now and then for recreational or motivational purposes is hardly sufficient to significantly transform neither the general practices of the classroom in question nor the general views of mathematics and its teaching and learning on the part of the participating teachers and students.

The problem of lack of time is often considered as more serious the higher into the grades you get. We have anecdotal evidence to suggest that this may also be the case for mathematics in Norway, not least as preparing for the exam appears to become a still more pressing issue. If this is so, it suggests that the potential influence of KappAbel grows if the competition is changed so as to take place in grade 7 or 8 rather than in grade 9. This may especially be so for the teachers for whom time is a constraint in the sense that if the pressure is eased, they will seriously consider taking part in the competition.

The respondents of the second group are concerned with what they consider a lack of compatibility between on the one hand the difficulty of the problems and projects of KappAbel and on the other the students and classes with which they

[3] We should repeat that for us this is not a value judgement. There is no connotation in the above statement that teachers should be committed to the KappAbel priorities. We are merely trying to make sense of what the reference to time as a constraint may mean.*

work. For some of these teachers this issue is primarily concerned with the organisational requirements of KappAbel. If you teach in a countryside school with grade-9 classes of only 5-10 students, it is difficult to compete with others that have many more students in the class, when the class is to participate as a whole. This becomes even more of a problem if the ambition is to move on to do the project and participate in the national semi-finals, because the requirement of the class being represented by two boys and two girls is difficult to meet. This problem is hard to solve, when a main concern for KappAbel is insert a stronger element of collaboration in mathematics classrooms and to avoid problems of gender inequity. Other teachers in this group see the main problem as being the level of difficulty of the tasks when viewed in relation to the students they teach, even if they work with average-sized classes.

The perceived problem of an imbalance between the requirements of KappAbel and the capabilities of classes and students in grade 9 seriously challenges the KappAbel-intention of being inclusive. This is so also if the majority of the students and classes in question would in fact benefit from participating. If teachers argue against participation on the grounds that their classes are too small or that the tasks are too difficult, then these classes and students are obviously not allowed to participate. It may be expected that this is the case to greater extent in countryside schools or in areas that are relatively more hit by social problems than in other areas. In turn this suggests that the perceived imbalance between requirements and student capabilities leads to a social imbalance between those, who participate and those who do not. This problem amounts to whether the inclusive intentions of KappAbel within classes have adverse effect on the problem of inclusiveness between classes. We have no suggestions for how the problem may be addressed.

The last group of responses is concerned with lack of information and knowledge about KappAbel. 23 teachers, ca. 10% of those who wrote qualitative responses, claim that this is a problem. Like with the two other groups, the comments on lack of information about KappAbel are made by teachers who vary significantly in their approach to KappAbel. Some know next to nothing about KappAbel, because they have decided against participating in it even without trying to get to know; others are much more likely to seriously consider the question of participation, if they have more information.

There seem to be two conclusions from this discussion. One is that KappAbel may be able to attract a few more classes into the competition or at least into doing the project, if information about it, including about the theme of the project, is available before the summer holidays. This would allow schools and teachers who make their plans for the following school year in May or June to participate. The other conclusion relates to the chosen ignorance of KappAbel on the part of some of the respondents. Although this may be interpreted as a very individual choice on the part of the teachers in question, it may also be seen as an expression of the social fabric of the institutional setting in which they work. In both interpretations it points to limitations of the potential of any educational initiative to fuse with the existing social and educational order of schools and classrooms. It points to setting realistic ambitions with regard to what it is at all possible to achieve.

One final comment based on the response of one teacher in this group, Ole, should be mentioned, not least as it has implications for the interpretations that should be made of the teachers' responses in general. Ole claimed that mathematics classrooms should be quiet at all times. We took this comment to indicate that his reasons for not allowing his classes to participate were not exclusively related to school mathematics. The teachers were not asked to respond to a question about disciplinary issues in KappAbel or in school in general. Consequently we have no way of knowing to what extent Ole's concerns resemble those of his colleagues in this respect. And consequently his comments substantiate our worries about the design of the study (cf. chapter 3). We then asked ourselves about the extent to which studies like the present one may impose a set of questions and possible answers on the teachers and students in question. We used the notion of symbolic violence to make the point. Ole's response illustrates the problem to which we have found no solution. The only remedy we can suggest using questionnaires like TS1 is to interpret the responses with care: given this range of possible responses the teachers have responded in certain ways. This tells us little about what they would have said, had they instead been asked about for instance about the role of disciplinary problems for their participation in KappAbel.

4.3 Results from TS1 – a summary

The question whether participation in the KappAbel competition has the potential to influence the teachers' and students' views of mathematics and of mathematics education relates to how many classes and teachers participate in the competition. It also relates to the characteristics of the ones who do participate both in terms of a range of relatively objective variables and of the teachers' mathematical and educational priorities.

45% of the mathematics teachers of Norwegian grade 9 classes in the academic year 2004/5 responded to the first teacher survey, TS1. These teachers come from 54% of the schools teaching lower secondary level.

The main finding is that all but one of the variables (gender, age, education (college or university), mathematical background and location of the school) are not systematically related to participation in KappAbel. The only exception is the teachers' mathematical background, as measured in terms the length of their studies in mathematics: The more time spent on mathematics at college or university, the greater the propensity to take part in KappAbel.

The factor analysis of items in TS1 that were to capture the respondents' views of mathematics came up with three factors, indicating emphases on *patterns and investigations*, *structures and logical reasoning*, *tools and applications*. There is a small, but statistically significant, difference, as the teachers who are more interested in investigations have a slightly higher tendency to become involved in KappAbel than their colleagues with other priorities.

Teachers with very different mathematical and educational priorities have their students take part in the competition. This means that a first ground condition of possibility for broad impact of KappAbel is met. 95% of the teachers whose classes do participate, claim that it is in order to motivate the students, 91% that it is because the tasks are challenging and 88% that it is because KappAbel is collaborative. These are the most frequently mentioned reasons.

Similarly the teachers whose classes do not participate indicate their agreement or disagreement with seven possible reasons for not doing so. The most frequently mentioned reasons are that project work takes too much time in view of the learning potential and that there is no tradition of participation at the school (both 46%).

253 of the teachers responded qualitatively to the questions raised. The remarks made vary considerably. However, 70 respondents refer to lack of time as an issue in relation to KappAbel. They almost consistently state that as there are only three mathematics lessons a week in grade 9, there is too little time even without participation in activities like KappAbel. This argument makes sense only if KappAbel is considered an activity that does not contribute significantly to the students learning of mathematics.

Chapter 5

Meeting teachers and students in KappAbel

The overall purpose of the teacher survey was to put into perspective the qualitative data from the small and heterogeneous material (questionnaire, interviews and observation) which we will present in this chapter. We have got some understanding of who the participating teachers are, both with regard to their objective characteristics (education, teaching experience, sex, etc.); to why they and their students participate in KappAbel; and to whether they consider themselves in line with the rhetoric of the current reform. Now we will meet a small group of teachers and some of their students.

In the quantitative study, Teacher Survey 1 (TS1), the population was all teachers teaching grade 9 mathematics in the academic year 2004/05. In the qualitative studies, the population was a subset of this group of teachers, namely all teachers who participated in KappAbel round 2 and planned to do the project work on “Mathematics and the Human Body”. We were in contact with 16 of these teachers and 14 of them returned the questionnaire, TS2. We interviewed seven of the teachers, who were all in the semi-final. We conducted interviews with five groups of students and made observations in four classes (one of these as a pilot during the KappAbel project work). Finally we had KappAbel reports from seven classes (see table 5.1).

Table 5.1 Overview of data from seven teachers and their students

Teacher interview	KappAbel Report	Student interview + report	Student interview + report + observation
7	7	5	2

Chapter 5 presents the results from the qualitative investigations in the research project. The first meeting with the teachers is framed by the questionnaire of TS2 (see appendix 2.1). The report from the second meeting with seven of the teachers and their students stems from interviews and KappAbel reports. In chapter 6, we will meet one of the teachers, her students and their mathematics classroom practices in a case study based on observations and interviews.

5. 1 Getting in contact

An email was sent in February-March 2005, to all 351 teachers who participated in KappAbel round 2. The main purpose was to get in contact with the teachers whose classes planned to do the project work on “Mathematics and the Human Body”, as we wanted them to respond to the second questionnaire, TS2. 117 of the teachers responded, but only 13 (11%) said that they planned to do the project. This percentage is smaller than the 29% of the respondents to TS1 (December 2004) who participated in KappAbel in 2004/05 and who indicated that they would do the project work even if they didn’t pass to the semi-final.

However, we also asked the teachers questions concerning the significance of participating in KappAbel (see appendix 2.2). Slightly under half of the respondents (52) said yes to the question of if the participation in the KappAbel competition would influence mathematics teaching in their grade 9 classroom for the rest of the academic year. Given four alternatives, as to the type of influence,

- 42 or 81% of these teachers (36% of all those responding) ticked off the type of problems student are working with,
- 24 or 46% (21% of those responding) ticked off the way students are working (e.g. project),
- 26 or 50% (22% of those responding) ticked off the amount and type of group work,
- 43 or 83% (37% of those responding) ticked off the students’ attitudes towards mathematics.

That is, quite a few of the teachers thought that a new kind of problems would enter their classroom practices and that their students’ views of mathematics would change.

All 13 teachers planning to do the KappAbel project work accepted being contacted again by email, and we sent them questionnaire TS2. 10 of these teachers returned the questionnaire. During the semi-final at Arendal we got in contact with four more teachers who also accepted to give an interview. In all, then 14 teachers returned the questionnaire. Among these, one was from Greenland and one from Sweden, thus the next section on TS2 is based on 12 relevant questionnaires.

5.2 Who responded?

The first part of the TS2 questionnaire (questions 1-8) concerns the factual characteristics of the teachers, such as gender, age, educational background. This part is identical to the first part of TS1 (cf. appendix 2.1). The two next questions (question 9-10) concern the teachers' school mathematical priorities, and they are based on the corresponding questions in TS1 and reduced on the basis of the factor analysis (see chapter 4.1). The next three questions (question 11-13) focus on the teachers' ideas of a good mathematics teacher and a good mathematics student and on the practices of his/her own mathematics classroom.

Factual information

As mentioned above, the TS2 population consisted of teachers who had participated in the two first rounds of KappAbel and who planned to have their classes to do project work on "Mathematics and the Human Body". We received 12 relevant answers to TS2 from seven men and five women, working at schools in 11 counties (fylker) spread over all the regions of Norway[1] (see table 5.2). Seven of the schools are situated in the town and five in the countryside. The teachers are between 30 and 60 years old. Four of them are in their 40s. Six teachers are educated at teacher training college and six have a bachelor's degree. One of the teachers has 0 Credits in mathematics, three have 5 Credits, three have 10 Credits, two have 20 Credits, and three of the teachers have more than 20 Credits in mathematics. Three of the teachers have five years of experience in teaching mathematics; and the other nine have between 7 and 39 years of experience. In 2004/5 they teach between 3 and 16 lessons of mathematics a week. Three of the teachers teach only 3 lessons pr. week, while two teach more than 10 lessons. , which indicates that he/she is a central mathematics teacher at the school. Nine of the 12 teachers teach mathematics to only one grade 9 class. For five of the teachers this is their first time to participate in KappAbel. Only one of the 12 teachers agrees to seeing himself as a mathematician, while ten of them agree to regarding themselves as a mathematics teachers (question 8).

[1] 8 of these 12 teachers reached the KappAbel semi-finals.

Table 5.1 Teacher background and experience (number) in TS2

A. Background and experience

1. Gender: **female**: 5 **male**: 7

2. Year of birth: **-50:** 3 **51-60**: 2 **61-70**: 4 **70-** : 3

3.1 Education:
teacher training college: 6 **bachelor´s degree**: 6 **other**: 0

3.2 The college level maths component of your education corresponds to approximately:

None	1
1/4 year (5 Credits)	3
10-15 Credits	3
1 year (20 Credits)	2
20- Credits	3

4. I have taught/been teaching maths for _____ years.

0-4 years	0
5-9 years	5
10-19 years	2
20-29 years	2
30- years	3

5. What other subjects do you teach? *We have not handled these data.*

6.1 This school year I am teaching ___________ weekly lessons of maths.

0-3 lessons	3
4-6 lessons	4
7-9 lessons	3
10- lessons	2

6.2 How many Year 9 mathematics classes are you teaching this school year?

1 class	9
2 classes	2
3 classes	1
4 classes	0

7. My school is located in **an urban area**: 7 **a rural area**: 5

Table 5.1 shows that most categories of background and experience are present among the teachers participating in TS2. Compared to the TS1 teachers these teachers are a little younger, they have on the average more credits in mathematics and a high proportion have 5 to 9 years teaching experience in mathematics.

School mathematical priorities

Questions 9 and 10 are meant to capture aspects of the teachers' school mathematical priorities: what are the main purposes of school mathematics and what are the characteristics of good mathematics teaching. In question 9, the teachers were asked to assign a total of 40 points to four statements on school mathematical priorities (see table 5.3).

Table 5.3 The teachers' assignment of 40 points (min/max)

Mathematics as a school subject should put *heavy* emphasis on		**Point**	
		Min	max
9.1	letting students learn mathematics applicable to their everyday lives.	5	20
9.2	letting students learn how to be creative and inquisitive in mathematics.	5	20
9.3	letting students learn logical reasoning and how to see the inter-connectedness of mathematics.	5	13
9.4	letting students learn how to master basic skills.	5	20
	In total	**40**	

We see the distribution (10,10,10,10) as an expression of equal priority to the four objectives formulated in items 9.1 to 9.4. This distribution we find in questionnaire no. 3, 7, 8 and 12, which are identical[2].

The questionnaires no. 1, 2, 13 and 14 are similar on two points (16,6,8,10); (15,5,7,13); (17,7,6,10); and (20,10,5,5) – high priority of mathematics to be used in everyday life and low priority of creativity and logic. The last priority is

[2] The questionnaires are numbered 1 to 14. Questionnaire no. 4 and 6 were rejected because they were not relevant (a teacher from Sweden and one from Greenland). The questionnaires no. 03, 07, 10, 11, 12, 13 and 14 are received from the seven teachers who are also interviewed.

also made in question no. 11 (10,5,5,20), but he gives a high priority to basic skills.

The questionnaires no. 5 and 10 are similar (10,15,10,5) and (9,13,13,5) – high priority to creativity and low priority to basic skills. Also questionnaire no. 9 (5,20,10,5) has a high priority to creativity but – on the other hand – she gives low priority to everyday mathematics.

As you may see in figure 5.3, 5 is the minimum points and 20 the maximum given to creativity and basic skills. Logical reasoning has 13 points max.

In question 10, the teachers were asked to assign a total of 30 points to three statements on school mathematical priorities (see table 5.4).

Table 5.4 The teachers' assignment of 30 points (min/max)

It is a characteristic trait of particularly *good* mathematics teaching		**Point**	
		Min	max
10.1	the students discuss, co-operate, and conduct mathematical investigations.	5	15
10.2	the students attain routine proficiency in the subject's methods and skills.	5	20
10.3	the students work on projects that include subjects in addition to mathematics (inter-disciplinary projects).	5	10
	In total	**30**	

We see the distribution (10,10,10) as an expression of equal priority to the three characteristic traits of good mathematics teaching formulated in items 10.1 to 10.3. We do not find this distribution in the questionnaires. Questionnaire no. 13 is the most even with a distribution of (12,11,7).

The responses in questionnaires no. 1, 5, 9 and 12 are identical: (15,10,5). They give priority to co-operation and investigative activities. The same goes for questionnaires no. 10 and 14 (15,5,10), but they give low priority to routine proficiency.

The questionnaires no. 3 and 8 are also identical: (5,20,5). Both give very high priority to routine proficiency. So do questionnaires no. 2, 7 and 11, although to a lesser extent: (7,15,8), (5,15,10) and (10,15,5).

As you may see in figure 5.4, 5 is the minimum points and 20 the maximum, which is only given to routines. The project work is not a high priority with a maximum score of 10 points. This is noticeable considering that all these teachers plan to do the KappAbel project.

Three of these teachers (05, 08 and 09) only responded to the closed questions (except a few interesting comments) and they are not included in the qualitative analysis below. Two of the teachers (01 and 02) were not interviewed and they are also not part of the qualitative analysis. Their comments, however, are included as illustrations. In what follows, we outline the results that are not included in the qualitative analysis in chapter 5.4.

In question 11, seven statements were given to characterise the particular skills of a good mathematics teacher, and the teachers were requested to think of a colleague whom they consider a good mathematics teacher: What is it that he or she masters to a greater extent than other colleagues whom you consider a less capable mathematics teacher? The teachers were requested to assess the statements in question 11 by circling a number: -2 means Strongly Disagree, -1 means Disagree, 0 means Undecided, 1 means Agree, and 2 means Strongly Agree (see table 5.5).

The two top scores are the ones of *Creating an atmosphere where the students discuss each others' suggestions and methods* (item 11.3) and *Linking mathematics to the students' everyday lives and to the the world around us* (item 11.7). The two lowest scoring are the ones of *Explaining the methods to be used by the students, so that they do not spend too much time on trial-and-error* (item 11.5) and *Engaging in and understanding difficult mathematics* (item 11.6).

In relation to item 11.1, one of the teachers made the following comment:

> 01: Students differ. Some need short, concise answers, while others need to conduct practical investigations.

When asked to describe characteristics of the last really good mathematics lessons of their own (question 12), most teachers talk about high student activity and engagement:

> 01: A great degree of student activity!

02: A good lesson is when I see that the students are having a good time doing mathematics. It may be that they become eager because they are succeeding, or it may be someone showing a method of calculation on the blackboard in the middle of a discussion that involves everyone, or they may be excited by a mathematical game. Largely, when I feel that learning is taking place on their terms.

Table 5.5 Characteristics of the good mathematics teacher (question 11)

A good mathematics teacher is *particularly* skilled at		Disagree - Agree				
		-2	-1	0	1	2
11.1	giving short and concise answers and explanations to students' questions.	0	0	3	2	7
11.2	using a variety of teaching methods.	0	0	0	6	6
11.3	creating an atmosphere where the students discuss each others' suggestions and methods.	0	0	1	1	10
11.4	getting the students started at exploring mathematical problems on their own.	0	0	2	6	4
11.5	explaining the methods to be used by the students, so that they do not spend too much time on trial-and-error.	1	3	2	3	2
11.6	engaging in and understanding difficult mathematics.	1	3	4	3	1
11.7	linking mathematics to the students' everyday lives and to the the world around us.	0	0	0	2	10

In contrast to this, the teachers in general describe the bad mathematics lesson as a practice with a high level of teacher activity:

01: The teacher lectures, and no-one pays attention or asks any questions.

02: A poor lesson is when I have to go through a difficult topic and some students scream out their frustration because they do not understand, and a negative atmosphere takes hold in the class. (Some students are negative towards anything that is new!)

One of the teachers, however, goes against the tide. He characterises the really good mathematics lesson like this:

> 08: Going through the topic material and then doing practical exercises based on the presented topic.

And he describes a bad mathematics lesson as one in which

> 08: The students are too focussed on the week's work schedule[3]. Doing exercises without going through anything in class.

When asked: What do you consider the most difficult aspect(s) of teaching mathematics? (item 12.3), he wrote:

> 08: The most difficult aspect nowadays is to gain acceptance that is necessary for the teacher to go through the topic before [the students] rehearse it by doing exercises. The week's work schedule being uppermost is one thing, another is that one is considered "old-fashioned" these days if one spends much time on presenting new topics. And then there is this differentiation. There should be more room for dividing-up/teaching in groups.

Two other teachers in very different way mention the need to cater for students of diverse abilities:

> 01: Reaching all of the students. We lose far too many.

> 02: Being able to offer difficult and challenging tasks to my best students. I feel that I give them far too little of my time. The weak and noisy students swallow me up completely, skin and all! I wish there were room for more streaming in mathematics – weak, average, strong – and that there were enough resources to have someone extra come in and help.

When asked what aspect(s) of mathematics teaching they found the most enjoyable (item 12.4) the same three teachers said:

> 01: When the students discover something new, or simply when they are pleased with themselves.

> 02: Variation is what makes teaching mathematics fun. The topics, the days and the lessons differ, and when the students ”see the light”, I know I have succeeded.

> 08: The subject is fun in itself, but of course it is fun when students cope!

In question 13, the teachers were asked to think of a student, whom they consider competent in mathematics and respond to a range of possible ways in which he or she differs from other students, who are less successful in the subject (cf. table 5.6).

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[3] Every week – or every second week – the students get a work schedule with tasks to be done and text to be read.

Table 5.6 Characteristics of the good mathematics student (question 13)

A successful mathematics student is *particularly* good at		Disagree - Agree				
		-2	-1	0	1	2
13.1	solving tasks quickly and correctly	0	0	2	6	4
13.2	finding their own way of solving a problem.	0	0	1	5	6
13.3	understanding when mathematics can be applied, and managing to apply the subject correctly.	0	0	0	3	9
13.4	remembering rules and facts (e.g. the tables).	0	1	2	6	3
13.5	asking new mathematical questions on the basis of the answers they have already found.	0	0	0	3	9
13.6	explaining their reasoning using everyday language and mathematical terms.	0	0	2	2	8

The two most highly favoured items are *Understanding when mathematics can be applied, and managing to apply the subject correctly* (item 13.3) and *Asking new mathematical questions on the basis of the answers they have already found* (item 13.5). The lowest scoring statement is *Remembering rules and facts e.g. the tables* (item 13.4). One of the teachers gave this supplementary comment:

> 02: Good students tend to have understood the connections in mathematics. This means that I can discuss with them and give them little hints so that they can cope with more and more difficult tasks. Good students also tend to get “hung up on” tasks and are adamant that they are going to succeed at solving them. It is fun to see them so enthusiastic.

So far we have discussed the school mathematical priorities of 12 teachers who responded to TS2 in fairly general terms. In the following section, we shall focus on KappAbel and the three basic ideas in the competition: why did these 12 teachers prioritize to participate in the KappAbel competition?

5.3 KappAbel and the three basic ideas

This section reports on the participation of the 12 teachers and their students in KappAbel. It addresses organisational as well as conceptual issues: Why did they participate and how did they organise the work in the first qualifying round? And what are the teachers' attitudes to the three basic ideas of collaboration, project work and gender equity, and how do they characterise their experiences with these ideas in the mathematics classroom, in general and in particular when involved in KappAbel?

There are items in the TS2 questionnaire that request teachers to provide reasons for their participation in KappAbel (question 15). These are identical to the corresponding items in TS1 (question 14), except for items 15.8 and 15.9 which are inspired by the teachers' comments in TS1.

Table 5.7 Why participate in KappAbel (question 15)

My class(es) is (are) participating in KappAbel		Disagree - Agree				
		-2	-1	0	1	2
15.1	in order to stimulate student interest in mathematics.	0	0	0	5	7
15.2	because the students wanted to participate.	1	0	3	4	4
15.3	because the tasks are challenging.	0	1	1	5	4
15.4	because my school has a tradition of participating.	4	2	3	1	2
15.5	because the competition offers opportunities for co-operation.	0	0	3	1	7
15.6	because this year's topic of Mathematics and the Human Body is a source of inspiration for engaging in mathematical activities.	2	1	8	1	0
15.7	because the competitive format may offer an opportunity for some students to take a more active part.	0	0	1	4	7
15.8	because the students like to compete.	0	0	3	3	6
15.9	because the students are motivated by the prize money.	3	1	3	3	1

The 12 teachers' response to question 15 correspond in general to the response to TS1, on this issue. The teachers choose to participate *In order to stimulate student interest in mathematics*, *Because the tasks are challenging*, and *Because the competition offers opportunities for co-operation* (see table 5.7). In TS2 nobody disagrees with these and in TS1 88-95% agree at least to some extent.

It is also worth noting that the reason of "this year's topic on Mathematics and the Human Body is a source of inspiration for engaging in mathematical activities" is not important in either survey. Just a single teacher in TS2 agrees and in TS1 only than 14% agreed that the theme was a reason for participating. As one of the respondents to TS2 phrased it: "The theme of the year didn't mean anything. We would have participated no matter what the theme was." (01)

The KappAbel homepage recommends that teachers and students prepare for the qualifying rounds for example by solving tasks from previous years. In order to get an impression of the classes' prior engagement with KappAbel and of their ways of organising the work we included question 16 on participation in KappAbel round 1 (see table 5.8). This was also a way to assess, whether KappAbel was an isolated incident or more integrated in the mathematics classroom practices.

Table 5.8 Organisation of the work in round 1 of KappAbel (question 16)

On your participation in KappAbel, round 1		Yes	No	Unde-cided
16.1	The students prepared to take part in the first round, e.g. by solving tasks from previous years.	5	7	
16.2	The students continued to work on solving tasks after they received the result.	4	7	1
16.3	The students were organised into groups, and each group worked on solving all the tasks.	9	3	
16.4	The students were organised into groups. Each group worked on solving some of the tasks, according to a distribution of tasks agreed upon in advance.	7	5	
16.5	The students worked on solving the tasks individually or in pairs.	3	9	

Seven of the twelve classes didn’t prepare to take part in the first round. Among the nine classes who reached the semi-final, five did not prepare for round 1. It is notable that the two classes that reached the national final did prepare for the qualifying rounds and continued to work on the KappAbel tasks after they received the results.

The classes organised the problem solving activities differently. In nine of the classes, the students were organised into groups, and each group tried to solve all the tasks. An apparent contradiction arises as seven teachers state that each group worked on solving some of the tasks, according to a distribution of tasks agreed upon in advance (item 16.4). Among these, four teachers had also given a positive answer to item 16.3. This reflects that some classes changed their tactic on the way; some groups also started with the first tasks and some with the last to ensure that they would solve all the problems. The three teachers stating that the students worked individually or in pairs (item 16.5) clarified by telling that some of the students worked in pairs. Thus – according to the teachers – no students worked alone in round 1.

Three teachers elaborated on their classes’ participation in the qualifying rounds as a response to an open item (item 16.6). You get the impression of a bustling activity:

> 01: They started out in groups, and aimed to solve all of the tasks. A little bit of competition at the start to find the answers. In many of the groups, they ended up working in pairs (or threes) on one task each, pursuing a variety of possible solutions. Finally, there was all-out comparison and co-operation in order to arrive at the result to be submitted from the class. -- Still, it needs to be said that many students were disappointed that the tasks were far too difficult, and they gave up pretty soon. On the other hand, those who did not give up worked really well and faced good challenges.

> 03: All the groups solved all the tasks, but half the class started with task number 5, so that they were certain they could finish. When a group found an answer, they wrote it on the blackboard (divided into 8 sections, like the sheet of paper), and when several groups arrive at the same answer it begins to look good, and then it ends up with the whole bunch working on one or two tasks they cannot quite agree on.

> 10: The students worked in groups of four. During the work, some formed pairs internally within the group, but the group was still central as the arena for discussion. Towards the end of the 100 minutes, the students circulated between the groups, arguing for their views and discussing the tasks they disagreed about (we had short plenary sessions where each group explained the answer they had arrived

> at). Then they would for example write down all their different solutions (if the point was to find several) on the blackboard, so that they could all compare their answers and discuss them on that basis. It was great fun to experience how the students took charge.

As concluded in the last account, it is our impression that the students organised the work in the qualifying rounds.

In the next three subsections, we will present the 12 teachers' attitudes – based on their response to TS2 – to the three basic ideas in KappAbel: 1) The whole class collaborates and hands in a joint solutions. 2) The class is doing a project work with a given theme. 3) There are two boys and two girls at the final team.

Co-operation

In TS1 78% agree with the first principle in KappAbel on collaboration and handing in a joint solution; 16% are undecided; and 6% disagree (item 16.1). Among the women 85% find that it is a good idea. In TS2, all teachers agree with this idea of co-operation in the competition (item 17.1). In order to get an idea of the organisation of work in their mathematics classrooms, the teachers were requested to think about their work with grade 9 and to tick the appropriate column for each item to indicate how often they use the method described (see table 5.9).

Table 5.9 Organisation of the work in the mathematics classroom (question 19)

		Almost every lesson	Once or twice a week	1 to 3 times a month	More rarely
19.1	The students work individually	5	2	1	3
19.2	The students work in pairs	8	3	1	0
19.3	The students work in groups of three or more	3	2	3	4
19.4	The students work together as a class with the teacher or a student at the blackboard.	4	3	4	1

Assessed from the teachers' response on question 19 – as reported in table 5.9 – they use a variety of organisational formats in the mathematics classroom. However, if you look at the data behind this table you will find that each teacher has her/his favourites. Two of the teachers use all four formats in almost every lesson, while two other teachers are changing between two ways of working (the students work individually or in pairs) and they rarely use other forms of organisation. Two of the teachers prefer that the students work in pairs. Three teachers rarely let the students work individually, while four of the teachers never let the students work in groups of more than two.

In chapter 5.5, we shall discuss the notion of co-operation further. It becomes evident that working together for example in pairs is not necessarily the same as co-operating in the sense of ”reach[ing] a common result, solution or product” like in KappAbel.[4].

In items 20.1 and 20.2, the teachers were requested to share their views on the greatest strengths and weaknesses of co-operation and individual work in mathematics. The following comments illustrate the view on co-operation of two teachers:

> 01: I see almost only advantages, but the weak point must be if someone fails to find a good partner and does not feel safe.
>
> 02: Strengths: The students learn each others' methods and strategies. Some students are good at one thing, others are good at something else. – Weaknesses: The students may also adopt each others' misconceptions, and some students just ”hitch a ride” and do not contribute or learn anything.

The same teachers share views on individual work with mathematics:

> 01: One should have a go on one's own too, but the weaknesses are that one may work a lot and for a long time without really learning all that much.
>
> 02: The strengths: The students can work at their own pace. – Weaknesses: The students learn one way of solving a problem and fail to see that ”several roads lead to Rome.”

The teachers comment on both mathematical and social aspects of the different working processes.

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[4] In English there is a distinction between *co-operation* (in general: acting or working together for a common purpose; concrete: helping each other) and *collaboration* (working together to create or produce something) which we do not find in Danish (samarbejde) or Norwegian (samarbeid).

Project work

The second principle in KappAbel is that the students should be involved in a project on mathematics within an interdisciplinary or practical framework. In TS1 (item 16.2), 57% agree, 35% are undecided, 8 % disagree. In TS2 (item 21.1), 11 teachers agree with the idea and one is undecided. In the characteristics of particularly good mathematics teaching (question 10), “the students work on projects that include subjects other than mathematics” (item 10.3) does not have a high score (max 10 points, on average 5,8 points). Nevertheless 11 of 12 teachers have marked project work in KappAbel as a good idea. This might be a sign of the competition being regarded by the teachers as an extraordinary event and not a natural part of the mathematics classroom practices.

Like co-operation, the notion of project work may be interpreted differently, and there is not necessarily a common understanding of the term among the teachers. In KappAbel it is defined as a work based on a problem and a formulation of this problem (see chapter 1.3). But according to Torkildsen (2000), who has been engaged in the competition since 2000, it is relevant to ask if the classes are doing thematic work or mathematical project work in KappAbel. Hence, we do not know how the teachers understand and practice project work, except for the seven teachers who were later interviewed, when they comment on the basic idea in KappAbel that students should be involved in a project on mathematics within an interdisciplinary or a practical framework (TS2, item 21.2).

However nine of the eleven teachers find that it is also a good idea to work with mathematical projects in other situations than the KappAbel competition (question 22). These teachers were asked to indicate level of agreement with nine more items concerning project work (question 23), cf. table 5.10.

Table 5.10 Why is it a good idea to do mathematical projects? (question 23)

Working on mathematical projects is a good thing		Disagree - Agree				
		-2	-1	0	1	2
23.1	because the students get to use mathematical skills they have already acquired.	0	1	2	3	5
23.2	because it offers a good way for the students to learn new mathematical skills.	0	0	1	6	4
23.3	because this method has a very motivating effect on the students.	0	0	1	7	3
23.4	because the students can take part in deciding what to work on and how to work on it.	0	1	1	6	3
23.5	because the students may change their attitudes to, and their views of, what mathematics is about.	0	0	0	5	6
23.6	because the students get ample opportunity for talking about mathematics.	0	0	1	5	5
23.7	because every student in the class can take part according to their own ability, performing a role and taking on a responsibility suitable to their level of proficiency.	0	1	0	5	5
23.8	but the students will be given very precise mathematical tasks to work on in their projects.	1	1	5	3	1
23.9	but such projects should ideally be carried out in collaboration with other subjects.	1	0	5	3	2

As a reason for working on mathematical projects the highest scoring item (23.5) is the possible change of the students' attitudes to, and their views of, what mathematics is about. 10 of the 11 teachers also see project work as motivating, and as a good way for students to talk about mathematics and to learn new mathematical skills. Only four teachers agree that the students should be given very precise mathematical tasks to work on in their projects (item 23.8).

Gender equity

The opinions are more divided when it comes to the third principle in KappAbel on equal sex distribution on the teams representing the classes at the semi-final and the final. In TS1 (item 16.3) 56% agree that it is a good idea with two boys and two girls on each team, 34% are undecided, and 10% disagree. There are 65% of the women who agree with equal sex distribution against 49% of the men. For some teachers we have no information on why they disagree with this principle of two boys and two girls representing the class. However, from some of the teachers' final comments, we know that local contexts make it difficult to fulfil this requirement, as in some countryside schools the classes are very small: "At our school there are only 5 girls in grade 9. This makes it difficult …" (see chapter 4.2).

The teachers were asked to indicate their level of agreement with the following item: "In the KappAbel semifinal the teams will consist of two boys and two girls. Do you consider this requirement a good idea?" (item 25.1). Nine of the 12 teachers responded "yes", one teacher said "no" and two were "undecided". There seems to be more hesitation among these teachers towards the principle of gender equity compared to the two first principles on co-operation and project work.

The teachers were asked to elaborate on their views on the requirement of having two boys and two girls in the teams. These are some of the arguments made:

> Should have liked to let the most interested participate – disregarding sex – but we have to admit the boys too. (01, a female teacher in her 40s)

Because boys are just as good in mathematics as girls, and because this makes more easy the procedure of selecting participants from the class. It would have been unjust if the class should select four arbitrary students. Maybe, in some classes, there is a strong group of boys (or group of girls) who wishes to send off four classmates of their own sex. When it is decided that it is two boys and two girls then you prevent this kind of problem. (Randi (07), a female teacher in her late 30s)

> Equality? Moreover they often think differently and the team will be more powerful. (08, male teacher in his 50s)

> OK with equal sex distribution; otherwise the boys would dominate.
> (09, female teacher in her 60s)

> Naturally there should be equal opportunities for the two sex to participate; otherwise there has to be two competitions – one for boys and one for girls. (Ola (12), male teacher in this 40s)

More arguments will shed some light on the teachers' experiences with whether gender matters in the mathematics classroom. But first we will look at the teachers' respond to items concerning perceived gender differences.

Differences between boys and girls

The teachers were asked consider the relative strengths of girls and boys with respect to a number of concrete aspects of the school mathematics (se tabel 5.11).

Table 5.11 Teachers' experience of differences between boys and girls

		boys	**girls**	**equal**	**unde-cided**
27.1	Showing interest for mathematics	2	3	7	0
27.2	Making an effort to get good marks	0	**7**	4	1
27.3	Making an effort to get praise	1	1	8	2
27.4	Co-operation	0	2	9	1
27.5	Competition	4	2	5	1
27.6	Deeper understanding	2	3	6	1
27.7	Practising skills	0	**5**	6	1
27.8	Working with numbers	1	0	10	1
27.9	Working with geometry	3	2	6	1
27.10	Working with functions	0	3	6	3
27.11	Working with probability	0	1	8	3
27.12	Confidence in their own skills in and knowledge of mathematics	**8**	0	4	0
	In total	21	29	79	15

Seven of the teachers claim that boys and girls are equally interested in mathematics (item 27.1), and eight state that the boys' self confidence is stronger than girls in the subject, while none claim that the opposite is the case. These results remind us of the result in KIMS, where 100 Norwegian mathematics teachers at grade 9 were asked similar questions (Streitlien, Wiik and Brekke, 2001). In a representative Finnish survey on understanding and self-confidence in mathematics, pupils in grades 5 and 7 reacted to statements like "I am not the type who is good in mathematics." and "I trust in myself in mathematics". The questionnaire also contained statements on success orientation like "I prepare my self carefully for the tests" and "For me, the most important in learning mathematics is to understand", and statements on defence orientation like "In mathematics one doesn't need to understand everything, when one only gets good marks in tests" and "I fear to embarrass myself in mathematics class." According to this study, boys seem to express stronger self-confidence in mathematics than girls. Weak pupils (boys and girls) had the weakest self-confidence while strong pupils had the strongest. However, the gender difference in alleged self-confidence stayed when comparing the most skilful girls and boys with each other (Hannula, 2005).

Gender differences in self-confidence were also part of the PISA 2003 investigation (Kjærnsli et al., 2004). The Norwegian girls expressed lower self-confidence both in relation to concrete tasks and in mathematics in general than the boys' even though their mathematical scores were almost equal.

Eight teachers ticked off that boys stand out in self-confidence in TS2 (item 27.12) and no one ticked off "girls". Two other items on gender differences differs from the others in that no one has ticked off "boys" and a group of five to seven teachers has ticked of "girls". These are item 27.2 (Making an effort to get good marks) and item 27.7 (Practising skills).

In what follows, we have added teachers' comments on gender and mathematics from the interviews to the arguments from TS2.

Gender matters in mathematics

Arne, a male teacher in his 30s, thinks that gender plays a role in mathematics. He experiences that boys are more interested than girls and that boys have more confidence in their own skills and knowledge. Another young male teacher, Steinar, believes that boys and girls often think differently, and he finds it valuable that girls get to be in the limelight too as in KappAbel. He also finds that it is important that everyone discovers that the girls are at least as good at mathematics as the boys are. He doesn't think that it makes much difference if you co-operate with girls or boys, but he points to a series of differences where the boys stand out more: interest for mathematics, competition, deeper understanding, working with geometry, and finally self-confidence in their own skills and knowledge of mathematics. Also in the interview, Steinar, pointed to gender differences:

> … My impression of boys is that they want to do everything fast, not as in haste makes waste, but very fast. While girls tend to contemplate more, or my girls contemplate the tasks more, and need to be convinced a bit by those boys. … So I think it's a good mixture to have both genders, because boys and girls tend to think slightly differently, I think. So I find that it's been very positive that they've been integrated in that sense. (l. 312- 316)

Gender differences in the statement of Steinar is formulated in the discourse of "mathematics as a male domain" like the fictive teacher, Kjersti's statement in item 26.1: "It is important that everybody gets to discover that girls are at least as good as boys at mathematics. It is therefore a good idea to have mixed gender groups." Peder (a male teacher in his 60s) also formulates his comments to this statement within this discourse: "Unnecessary – it is obvious to everyone that the girls are at least as good [as the boys] at maths." However, it is possible to interpret Randi's (a female teacher in her 40s) reason for finding equal sex distribution in the teams as a good idea - as an indirect critique of this dicourse: "Because boys are as good as girls at this subject, and because it makes the selection process easier (...)." In the "mathematics as a male domain" discourse it is assumed that boys are the best just like in the early investigations of gender and mathematics (see Brandell et al., 2004). Although this is not the case in lower secondary school mathematics, Randi is the only one of the teachers who has a critical comment although it is just implicitly. Another female teacher just formulated it like this when arguing that the requirement of equal sex distribution in the KappAbel teams is not a good idea:

> In my classes, I have very able and mature girls, the boys are a little stuck both in mathematics and otherwise. The girls found it stupid that they had to leave with "not so good boys". The boys are also much more vocal and – unfortunately – the girls have a tendency to listen to them even if they are wrong and the girls are right. If we had reached the semi-final I would have preferred to put together the teams in a way fitting our class." (02, A female teacher in her 40s)

Øystein, a male teacher in his 30s, was undecided about the gender requirement:

> Socially, it's a good idea, but not in terms of the subject of mathematics. It may seem as if girls are more geared towards the theoretical, whereas the boys are more practical, but the tendency is not so pronounced that both genders HAVE TO be in a group. (TS2, item 25.2)

During the interview, Øystein elaborated on these remarks about gender difference when the interviewer asked how he feels about boys and girls working together in the semi-final:

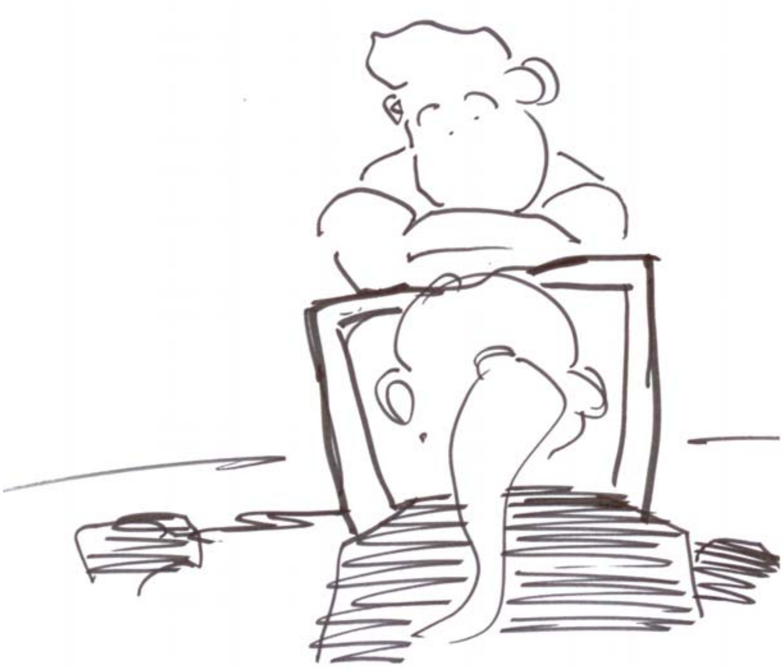
> Well, I think that's good. Boys and girls think differently. At least my boys and girls. It's the boys who ... the boys in my class, they're often from a farm, because this is a farming community. They've been working on the farm since they were six, right. So they're very practical-minded, and they see this stuff about ... geometry for instance, rather quickly. This stuff about spatial geometry and that kind of thing, and sort of purely logical things they see fairly well. While the girls, they may be a bit more ... a bit more technical. Even though these girls here are fairly practical, they're still ... They have a bit more patience. They are more able to sit and work on tasks for a while, whereas the boys, they take an answer for an answer sort of. Yes. (l. 433-441)

To the question about the KappAbel principle that the final team must have two boys and two girls, Øystein responds that it was the best four who participated this year – in other words not a problem, but when they participated in the final in previous years there were perhaps six or seven girls who should have been picked before the first boy: ”The boys, they had to sort of ... study to catch up sufficiently to provide a couple who could make a contribution.”

"The quiet girls" has often been a theme in the debate on gender in the schools. In item 26.4 the fictive teacher Svein claims: "Girls may find it difficult to make themselves heard in mathematics. It is therefore a good idea to have boys-only and girls-only groups." Three of the teachers comment this statement by saying that it is the teacher's responsibility that the girls make themselves heard. One of them expresses it like this:

> … And the girls don't have any problems to make themselves heard in my lessons. After all it is me who control who has the floor during the lessons!
> (07, female teacher in her 40s)

Two other teachers stress that the girls are definitely not mute: "I totally disagree with Svein; girls speak out just as much " (11, male teacher in his 60s).



Gender doesn’t matter in mathematics but …

Most of the teachers taking a general position on the theme during the interview or in TS2 do not think that gender matters in the mathematics classroom. However, when it comes to specific examples some of them, like Ola a male teacher in his 40s, comment that boys have more confidence, and the girls have a deeper understanding in mathematics. Discussing gender and KappAbel in the interview, Ola said that he had never thought about this, but the girls tend to be more mature, they tend to have a greater sense of responsibility. He quoted one of the girls: ”We really have to try, we really have to do something.” And he continued: “So that come-to-mommy role, I’ve seen that all right. I do, after all, have just as many girls among the best as I have boys…” (l. 666-668)

According to the teachers’ comments and accounts, gender matters in these 12 mathematics classrooms. Most investigations – our own included – ends up with the conclusion that boys in mathematics express more confidence in their own skills and knowledge compared to girls. We would like to temper any conclusions on this matter with the observation that since mathematics traditionally is seen as a male domain, it might be more threatening for a boy to admit – even to himself – that he is not good in mathematics, and this might influence the way boys and girls talk about their own proficiency in mathematics. We also note that the teachers are more in agreement with “mathematics as a male domain” when they talk about boys and girls in general, whereas their opinions are more complex and even-handed when they talk about their “own” boys and girls.

5.4 First meeting with six teachers

In this section, we give short presentations of six mathematics teachers based on their response to the questionnaire, TS2. In the next section, we shall elaborate on these teachers’ school mathematical priorities and their engagement in KappAbel, based on interviews conducted with them. Our analysis is structured by two central issues addressed in the rules of the didactical contract: (1) What is mathematics and mathematics education? (2) How do you learn mathematics? (See chapter 2.3). Our first meeting with the seventh teacher we interviewed, Kristin, is reported in chapter 6 as a part of the case study of her and students.

Arne

Arne is a teacher in his 30s, who has eight years of teaching experience in mathematics. He has more than 20 Credits in mathematics, but he does not consider himself a mathematician. He emphasises creativity, investigative work and co-operative activity. Also he prioritises that the students learn to reason logically and that they see the interconnectedness of mathematics. He gives very low priority to basic skills and routines.

Mathematics and mathematics teaching

Arne’s basic attitude appears clearly in his idea of a good mathematics teacher. First of all, he or she is to create an atmosphere of discussion and investigation and relate mathematics to the everyday life of the students and the surrounding world. As a comment to the teacher’s ability to giving short and concise answers and explanations to students’ questions (item 11.1), he says that:

> even though there is something to be said for giving short and concise answers in some situations, there are other situations where the teachers should encourage wider discussions. But being able to answer in a short and concise manner without getting lost in longwinded and difficult trains of thought is of course a good quality to have. (question 11)

Arne gives the following characterisation of a poor mathematics lesson:

> Poorly prepared. The students do some exercises from the coursebook. We run out of time to discuss them. The tasks aren't challenging enough for the strong ones, and the "weak" ones are just completely lost. Many of them spend 15 minutes just getting started, and everyone is just longing for the lesson to be over. (Luckily, this has rarely been the case over the past year) (item 12.2)

Some of the most challenging aspects of teaching mathematics, Arne identifies thus:

> Having a satisfactory overview of and an overall strategy for the subject, so that I can make wise choices and priorities – looking for contexts where a topic can be brought in/support another one. I suppose it'll come with more experience…
>
> It is also a challenge that the curriculum is so comprehensive in relation to the lessons allotted to the subject, and that the (good?) processes I want to stimulate take TIME. (item 12.3)

Arne's process orientation is evident in these quotations. The last one also indicates the dilemmas he faces in his everyday teaching.

Learning mathematics

Co-operation and discussion are essential to Arne when it comes to learning mathematics. His students seldom work alone, but work in pairs nearly every lesson and in bigger groups once or twice a week. According to Arne, co-operation adds to the proficiency of the students and contributes to their conceptual development as well as to strategic skills. Moreover, Arne considers co-operation an important aspect and an integrated part of mathematics as a school subject:

> (...) co-operative skills need to be integrated into the work that is done in the subject, rather than being regarded as a separate area – just as Excel is applied to problems within all parts of mathematics as a school subject, rather than following the "course model", where we do Excel for 2 weeks. This applies to other subjects than mathematics as well: Co-operation is connected to work in the subject that needs to be done anyway. (item 18.2)

In the questionnaire a teacher, Oscar, is quoted for a statement to the effect that dialogue among students tends to be about anything other than the subject unless their co-operative activity is directed towards small, precisely defined tasks, Arne says:

> Nonsense. Besides, students working individually also use their energy on anything but the subject. On dreaming, for instance. (item 18.1)

He has little time for teacher-controlled co-operation: ”Let the students out – spring is here!” On item 11.5 of explaining to the students what methods they are to apply, Arne says that:

> there is something to be said for explaining the methods too, but I sense a basic attitude shining through – one which says that the students should always be supplied with a ready-to-use recipe before they're allowed to have a go. I have no confidence in this as a method, since an important goal as far as I'm concerned is to build the students' UNDERSTANDING. (question 11)

Interdisciplinary project work allows the students to learn new mathematics, but in particular it may challenge their views of the subject: "Because it [project work] can make the students see that mathematics is not an isolated subject but can be applied in other areas." Arne, however, also points to activities within the subject, for instance in the form of mathematical projects, as necessary.

KappAbel participation

Arne participated with one of his classes in 2004/05. Participation is a tradition at his school, and they participate among other things because the students want it. The theme of the year "Mathematics and the Human Body" was not important. They would have participated in no matter whatever the theme. About the money prize he notes: "I think it is a bad trend that Norwegian schools and students should be stimulated by money to make them bother to do a job and concentrate on developing mathematical competences." The students prepared for the first round by solving old KappAbel problems, and they continued working on the problems after having had the results. The students worked in groups who solved part of the problems as they had planned to do but they also circulated among the groups. Arne had a very positive experience of the students taking control of the process.

In general, Arne seems prepared to let the students take control in the classroom, and his views of mathematics and of teaching and of learning does not appear to be alien to those of KappAbel. In other terms, KappAbel may serve to consolidate rather than change his dominant school mathematical priorities.

Ola

Ola is a teacher in his 40s, who has more than 10 years of teaching experience in mathematics. He has 10 Credits in mathematics and does not consider himself a mathematician. He puts equal emphasis on the four learning objectives in question 9, i.e. the ones of applications, creativity, logical reasoning, and basic skills. Good mathematics teaching is to him characterised by discussion, co-operation and investigative activity.

Mathematics and mathematics teaching

According to Ola, a good mathematics teacher uses a variety of working methods: he gives short and concise answers and explanations to the students' questions, he creates an atmosphere where the students discuss each others' suggestions, and finally he relates mathematics to everyday life and the surrounding world. The good mathematics lesson is well prepared and characterised by the students' being eagerly involved in mathematical activity. Also it is characterised by teacher-student and student-student interaction in whole class as well as in small group settings. Ola thinks that the most difficult part of teaching mathematics is to generate engagement and interest in the subject. Mathematics is not his favourite subject.

Ola agrees that interdisciplinary projects involving mathematics are a good idea. He agrees with all the nine reasons listed in question 23 for engaging in such projects, but especially that the students get ample opportunity to talk about mathematics (23.6) and that every student can find a suitable role for him- or herself in them (23.7). He also says that in such projects mathematics becomes ”the tool that it really is”. Projects should be done in co-operation with other school subjects, and Ola's idea of project work does not include giving the students exact mathematical problems to solve in the projects.

Learning mathematics

According to Ola, a good student of mathematics is one who can solve the problems quickly and correctly (13.1), who remembers the rules and the facts (13.4), and who understands when mathematics can be used and who knows how to use it (13.3).

As mentioned a teacher, Oscar, is quoted in the questionnaire for the statement that *Co-operation in mathematics should only be used for small and concisely formulated tasks. Otherwise, talk among the students more often than not tends to be about anything but the subject matter*. Ola comments that this is ”Wrong – students work well when the class has a good climate for learning.” (item 18.1) He finds that the students in grade 9 are good at taking responsibility, and he states that co-operation improves the students' mathematical competences, because they challenge each other's understandings and make use of their individual abilities. One or two times a week his students work in groups of more than two, but in nearly every lesson they work alone or the whole class together with Ola or with a student at the blackboard.

KappAbel participation

Ola has participated in KappAbel with one of his grade 9 classes in 2004/05. He wants to motivate the students particularly because of the competition, because it is an opportunity for some of the students to be more active, and because the students like to compete. The students had not prepared for the first round and did not continue working on the solutions after having had the results. They worked in groups of two or more and solved part of the problems as suggested by KappAbel.

In the questionnaire, Ola presents a fairly all encompassing view of school mathematics, including all aspects of the subject mentioned, but especially focussing on the tool aspect. Also his heavy emphasis on co-operation seems significant: this he claims, is the main method of work. The main problem he faces is the one of the students' lack of motivation. This is also a main reason why he has them participate in KappAbel.

Peder

Peder is a teacher in his 60s, who has 39 years of teaching experience in mathematics. He has more than 20 Credits in mathematics and he is undecided as to whether to consider himself a mathematician. His school mathematical top priorities are that students learn to master basic skills, and that instruction is organised in order that the students attain routine proficiency in the subject's methods and skills.

Mathematics and mathematics teaching

The good mathematics teacher is to Peder someone who has a wide range of methods to use. He is very aware that students are individuals who have different needs. This is also what he considers to be the most challenging aspect of teaching mathematics: "Great variations among the students in terms of ability and achievements, as well as attitudes."

Peder finds it important that the students learn mathematics that may be used in their everyday life, but he gives low priority to the creative and investigative dimension and to logical reasoning within the system of mathematics. Also, he does not prioritise working with projects that go beyond mathematics as a subject.

Learning mathematics

Mathematics is learned through the teacher's presentation in combination with individual student activities. Peder characterises a good mathematics lesson as one in which there is ”variation between ”successful” teaching and substantial activity among the students.” And a bad lesson is one that contains ”Too much (and 'not-taken-in') teaching. Little student activity, external things 'disrupting'. ”The students often work alone. However, he definitely considers co-operation (mostly in pairs) as a valuable working method, and he practises it in nearly every lesson. The students should be given initiative, and the teacher should not always be in control of problem solving, or the one who presents at the blackboard.

Peder does not consider project work something that furthers the learning of mathematics: ”It takes an awful lot of time, which I – as an old man – feel could have been spent in far better ways (OUCH!)”.

KappAbel participation

Peder had his two classes in KappAbel in 2004/05. This is not because of the traditions at his school, but in order to motivate the students. They prepared the first round by solving old KappAbel problems, and they solved the problems of the competition in groups of three or more. One of the classes moved on to the semi-final and therefore did the project work “Mathematics and the Human Body”. Considering Peder's view of time spent on projects and the low priority he gives to interdisciplinary project work, it is likely that the class only did the KappAbel project because they reached the semi-final.

Randi

Randi is a teacher in her 40s with 12 years of experience as a mathematics teacher. She is strongly engaged in science but not in mathematics, and she does not consider herself a mathematician. She emphasises equally the learning of everyday mathematics, creative and investigative mathematics, logical reasoning and interconnectedness of mathematics. At the same time she values highly the mathematics teaching where students acquire safe routines in the methods and skills of the subject.

Mathematics and teaching mathematics

A good mathematics teacher is according to Randi a teacher who uses a wide range of methods, but always presents the content in a well structured way. She says that variation is what characterises a good mathematics lesson:

> (...) The best lessons are the ones with variation:
>
> They often start with a presentation of a topic.
>
> I use the blackboard quite a bit for presenting theory, using lots of colours and illustrations. The students work with books containing their own compilation of rules.
>
> The students work individually or in groups, doing exercises.
>
> Finally, we discuss problems encountered by several of the students during the course of their work. (Here, it is they who explain to each other).
>
> When I have access to tools that illustrate what we are doing, I use these. (Geometry, volume, area, probability, statistics, coordinates, scales, equations, length/velocity/time, numerical systems etc.) When it is practicable to let the students do simple investigations, I allow them to do so. (item 12.1)

She uses all forms of co-operation (question 19) several times a week. It shows in from many of Randi's statements about her teaching that spoken communication and discussions about mathematics constitute a significant dimension in her approach:

> The mathematics lessons I can look back upon as successful are the ones where I have been well prepared. Even if the lessons are well prepared it does not mean that I manage to do what I had planned. Often, the students are so excited that we spend time discussing other mathematical questions. (item 12.1)

Randi does not think that interdisciplinary project work like in KappAbel is a method of work that should be applied in ordinary teaching. She lets the students (alone or in pairs) do small projects.

Learning mathematics

Each student learns at his or her own pace, and it is up to the teacher to involve them at their own level, so that mathematics can be fun even for the less successful ones:

> A good mathematics teacher is for me someone with a good knowledge of the subject, who at the same time is good at structuring the teaching. It is important to

start where the students are at in terms of their mathematical knowledge; students' knowledge varies a great deal, and one therefore has to take the time to begin with the simplest aspects of each topic. It is important that the students get to work at their own pace.

A safe classroom environment plays an important role in learning. A good teacher knows the strengths of each student. He or she can use this knowledge constructively to enhance the learning process of each individual student.

The result may be an increasing number of students who find that mathematics is a fun subject, even though not everybody is equally good at the subject. (question 11)

And it is precisely the differences among the students which Randi considers the most difficult aspect of teaching mathematics:

The most difficult aspect is to meet all of the students at their own level in terms of their mathematical understanding, and give each of them something to aim for. This is because they arrive with such different degrees of knowledge. Some students struggle with the basics of a topic, while others know the entire lower secondary school curriculum already, and yet others know even more...

It is important not to stay too long with what is easy (for the students who are good at mathematics) – and not to progress too fast (for the weaker ones). It is a mixed class – as it should be, but the fact that there is such great variation makes it difficult for me as a teacher to maintain the students' motivation and joy about the subject! (item 12.3)

In her view, the students who are best in mathematics are also ”the ones who are most likely to work in a determined manner with solving tasks.”

Randi sees it as important for the students' learning that they know how mathematics can be applied:

A good mathematics lesson is one where the students are with me in terms of the topic we are addressing. If this is the case, I often manage to show them how they can apply mathematical concepts to their daily lives. As a maths teacher you often get to hear: ”What do we have to learn this for? I can't be bothered learning something I won't have any use for later!” (I remember from my own days as a student that the mathematics teacher stuttered out an answer that my class did not buy.) (…) (item 12.1)

Randi also believes that feeling safe about their learning environment is important to the learning processes. This requires, among other things, good planning and that the students know what is going to happen.

Co-operation is advantageous both to students who are good at mathematics and to the weaker ones: ”I agree that students are able to challenge each others' understanding when working together. I have observed this particularly among those who are good at the subject.” And:

> For some students it is a must that others help them along the way. They need to be motivated to get started, and in order to maintain their concentration. For these students, co-operation may be the key to wanting to learn something. But it is equally important that these students are left undisturbed to work as soon as they have understood the mathematical issue. It may then be an advantage if they can work individually, with a teacher nearby to motivate them for further work. (item 18.4)

Randi believes that co-operation, with the fun it involves, facilitates learning:

> (...) those who wish to work on mathematical topics do so more efficiently if they are allowed to do it with someone else. It is more fun, and when something is fun, people work faster. If they devote time to small talk, this too will be conducive to their learning: They are having a good time while solving mathematical tasks (...). (item 18.1)

All in all, fun and joy are essential to her, but the individual student's own contemplation and individual practise are also important: ”It is not always necessary to discuss with others in order to learn mathematics. Much of it can be learned by individual contemplation. It takes practise to develop one's skills!”

She leaves the organisation of the groups to the students:

> I think groups are organised as the class want them. Some are girls-only, some are boys-only, and some are mixed. The ability to co-operate, as well as the willingness to do so, should guide the organisation of the groups! (item 26.2)

KappAbel participation

Randi participated with one of her classes in KappAbel in 2004/05. It was not because of tradition at her school, but to motivate the students and because of the co-operation and of the competitive element. They had not prepared for the first round. In the first round, the students formed groups that each worked on solving all the problems. In her final comment she elaborates on her experiences with the project:

> I have to say that I've never before let any of my classes do the KappAbel project. In the past, I have let my students do other small projects (individually, in pairs, or in groups).

> The whole class having to do one common project was a new experience for me! It was not as easy getting the class to carry it through without my interference. Luckily, the two student representatives[5] took on the main responsibility (while the four KappAbel representatives were appointed group leaders). The student representatives contacted me several times during the process and asked for my help in getting their classmates to do the work assigned to them.
>
> In a class there will always be some students who need to be followed closely by the teacher in order to get their work done. Without adult supervision, their unwillingness to work may be contagious, and it is a frustrating task for the students in charge to have to solve the conflict which may ensue.

Co-operation and discussion are central elements in Randi's view of school mathematics. She speaks of theory, tasks and problem solving, but problem solving in the KappAbel sense does not appear to be a part of it. Also, it does not have a high priority to work with bigger projects. When Randi's classes have previously participated in KA , they did not do the project, but now that the class is in the semi-final, they have to. Her experience, however, was that the students needed her assistance in order to do the project. One may speculate that this experience may be due to what appears to be her relatively prominent position as organiser and presenter in everyday mathematics instruction. It is not clear whether participation in KappAbel may contribute to Randi changing her view of the roles of the students and herself in the mathematics classroom.

Steinar

Steinar is a man in his 30s who has 5 years of experience with teaching mathematics. Mathematics is his favourite subject (20 Credits) and he sees himself as a mathematician. He prioritises teaching everyday mathematics. Good mathematics teaching is to him includes discussion and co-operation, routine work, and interdisciplinary project work, even though if he gives less importance to the latter (7 points).

Mathematics and mathematics teaching

Steinar finds that the good mathematics teacher should be able to give short and exact answers and explanations to students' questions, to create an atmosphere where the students discuss suggestions and methods, and to relate mathematics to the everyday life of the students and to the surrounding world.

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[5] During the project work, the class was divided in two groups.

Steinar associates a good mathematics lesson with the students ”being left alone to work undisturbed; discussion of mathematical topics with individual students and in the class as a whole”. A poor lesson is one characterised by ”Poor planning/structure. Students finished sooner than I had expected, resulting in dead time and noise”.

Steinar finds ”motivating everyone and demonstrating that all types of mathematics are useful” the most difficult aspects of teaching mathematics. Students often say “what do we need this for – it is not always equally easy to give them an answer.” However, he is otherwise happy with teaching mathematics, which is a subject he masters well himself. He does not, however, consider it necessary to engage with and understand difficult mathematical problems to be a good teacher. Steinar thinks it is fun to teach mathematics ”When I see students progress and cope. When a test shows that they do well on things I have emphasised”.

Steinar finds project work useful, primarily because it gives the students the opportunity to change their attitudes and their views of what constitutes mathematics: ”Seeing mathematics in a completely different light than exercise upon exercise. Seeing that most of the things we are surrounded by contain mathematical aspects.” (item 21.2)

Learning mathematics

Steinar believes that dialogue, conversation, is important to the learning of mathematics. Solving tasks individually is not enough. Emphasising pair work, he explains his reasons for believing that student co-operation is valuable as follows:

> It is good for them to ”talk” mathematics with others in order to expand their own understanding and to focus more on the subject. Not just to do exercises. (item 17.2)

That is why he agrees with Ole’s statement in the questionnaire (item 18.3): *Students learn mathematics by putting their thoughts into words. They should therefore co-operate and discuss with each other.* But Steinar also thinks that individual work is important, even though he rarely uses it. The advantage is that the students can work at their own pace; while it is a possible disadvantage that they do not realise that they have problems, but quickly move on to the next task. It is according to Steiner another positive aspect of co-operation that the teacher leaves some of his control of the teaching-learning process to the students.

KappAbel participation
Steinar participated in KappAbel with his grade 9 in 2004/05. The important reasons for this were that the students like competing, that competition gives some students the opportunity of being more active, and finally that the competition could motivate the students for learning mathematics. The students had neither prepared for the first round, nor continued working on the problems after having been given the results. They were divided in groups of three or more, and each group solved all the problems.

Co-operation and communication are central elements in Steinar's view of school mathematics. She speaks of theory, tasks and problem solving, but problem solving in the KappAbel sense does not appear to be a part of it. Also, it does not have a high priority to work with bigger projects. When Randi's classes have previously participated in KappAbel, they did not do the project, but now that the class is in the semi-final, they have to. Her experience, however, was that the students needed her assistance in order to do the project. One may speculate that this experience may be due to what appears to be her relatively prominent position as organiser and presenter in everyday mathematics instruction. It is not clear whether participation in KappAbel may contribute to Randi changing her view of the roles of the students and herself in the mathematics classroom.

Øystein

Øystein is a man in his 30s who has 6 years of experience as a mathematics teacher. Mathematics is his favourite subject (Credits >20) and he considers himself as a mathematician. He prioritises teaching the students mathematics that is useful in their everyday life. He gives only 5 points each to logical reasoning (9.3) and basic skills (9.4). When it comes to characterising good mathematics teaching, he favours discussion, co-operation and investigation. Here again Øystein attaches less importance to routine.

Mathematics and mathematics teaching
A good teacher is seen by Øystein as someone who can create an atmosphere in which the students discuss each other's suggestions and who can relate mathematics to everyday life and the surrounding world. He does not agree that the good teacher should explain the methods to the students, so that they do not spend too much time trying to find a solution. He characterises a really good mathematics lesson in his class as ”Eye-opening for the students” (item 12.1),

and a poor lesson as ”Routine” (item 12.2). He considers ”creativity” as both the most difficult aspect of teaching mathematics, and the most fun.

According to Øystein, a good mathematics student finds his own ways of solving a problem and asks new mathematical questions on the basis of the answers found. The good student understands when mathematics can be used and manages to use it correctly.

Øystein finds that far-reaching projects such as the ones in KappAbel are a good idea for the teaching of mathematics: ”Mathematics is present everywhere in society, so why not include several subjects/angles/areas?” (item 21.2). The students are motivated by project work, but what is most important is that they will have the opportunity of changing their conception of mathematics, and of talking about mathematics. This leads to Øystein’s basic view of how to learn mathematics.

Mathematics and learning mathematics

He describes co-operation as the best way of learning mathematics. Among the teachers’ statements he points out Ole’s as one that the other teachers can learn from: *The students learn mathematics by putting words to their thoughts themselves. That is why they have to co-operate and to discuss with one another*. (question 18). Øystein also agrees when Arne states that *The students can challenge each other’s understanding when working together. It is therefore important that they co-operate when learning mathematics* (item 18.5). In his opinion, ”one has to trust the students enough to let them set the terms of the co-operation. After all, they’ll soon be adults …” (item 18.6)

In responding to what methods he uses with his grade 9, Øystein states that his students work in groups of more than two in practically every lesson, while they work individually a couple of times a month. He describes the weaknesses associated with individual work as follows: ”It makes one anti-social, focussed only on technical matters, inflexible, too prone to quick conclusions.” (item 20.2)

Øystein participated with a class in KappAbel in 2004/05. This was not because of a tradition at his school or because of the prize, but because the students wanted to participate and because of the opportunity to co-operate. The students prepared for the first round by solving problems from earlier years, and they continued working on the solutions after they had the results. They were divided in groups which worked to solve all the problems.

Øystein and his students wanted to do something different in their mathematics classroom and they accepted the KappAbel invitation.

5.5 The second meeting with six of the teachers and their students

Six teachers were pictured by our interpretations of their response to the questionnaire (TS2) in chapter 5.4. In the light of this first meeting with the teachers, we now go deeper into the qualitative material: interviews with the teachers, group interviews with their students[6], and analyses of KappAbel reports. From the analysis we aim to obtain a more elaborate picture of the teacher's school mathematical priorities. And for the first time in this study we shall meet the students. Between them, the reports on the teachers and the students seek to contribute to an understanding of the potentials of KappAbel for changes in the views and practices of school mathematics. In our understanding of mathematics classroom practices and possible changes in these practices, the didactical contract is vital. This contract is negotiated and established by this specific teacher with these specific students and constituted in and by their mathematics classroom practices (see chapter 2.3). In what follows, we do not claim to have access to the specific character of the contracts so established. However, we do want to shed light on how the teachers and students address two issues related to the contract, when they are not in the classroom. These are the issues of "What is mathematics and mathematics education?" and "How do you learn mathematics?" A third issue ,"Why learn mathematics", was mentioned as an element of the didactical contract in chapter 2. This issue, however, was not addressed in the interviews, but only by question 9 of TS2 (see table 5.3). Thus, we do not address the so-called justification problem in this chapter.

Arne and his students

Arne is the 30 year old male teacher with eight years of teaching experience in mathematics, whom we met in chapter 5.4. He has more than 20 Credits in mathematics, but does not consider himself a mathematician. Through his answers in TS2 we get the impression that he emphasises creativity and investigative and co-operative activity. Also, he finds important that the students

[6] All interviews with teachers and students (except the teacher Peder) were made by Kjersti Wæge, the Norwegian research assistant.

learn to reason logically and to see the inter-connectedness of mathematics. He gives very low priority to the basic skills and routines. In this section we meet Arne again, and we are introduced to his students' views of mathematics, as they are expressed in the project of "Mathematics and the Human Body".

Mathematics is investigative activities and creativity

When Arne is asked to tell about a normal lesson, he says that the students work on some booklets that he has prepared with a colleague. A part of the problems in the booklet challenges the best students, so that they don't end up being a booklet ahead of the others.

> Arne: Well yes, it depends a bit on what type of task we're talking about, but this is when we tend to gather them. And then… I point to… / Let's say we've progressed a bit so that we have some variation with some having done two pages of the booklet and some having managed only a little bit on the first page.
>
> I: Mm
>
> Arne: Then I gather them, and … maybe on the video projector I've rigged up … we do a summary of some of the things they've done earlier. And that's where things were a bit too randomly organised because of people falling ill and all that. But this is how one wants to be able to do it. Either that one then takes/
>
> I usually save up an entire lesson when I've gone to the trouble of rigging the equipment, and then we spend the lesson summing up their earlier work and the experiences they've had. And then … when they proceed with their work, I try to tell … which tasks are, or which things it is important that they at least take a quick look at. And then eventually they have / they're really working with the booklet like … in … at their own pace, two and two together. Two between each computer to get a discussion, sort of. (l. 110-128)

The interviewer asks about the roles taken on by Arne and by his students, and Arne continues:

> Arne: Yes, then … I try to get them to … to tell us about what sort of things they've discovered. So (…) If we have some tasks maybe I've done a … prepared an example myself or I've picked out an example from the students that I've taken from the server. Brought it in and then we discuss it. So then it may be leading the discussion where they can make their own contributions. Or it may be to sort of ask a student if he or she can tell us about what they've done. (…) And sometimes it'll be that I go in and see something they're doing that is incorrect and then … or that I want to emphasise and then I explain. (l. 146-153)

The interviewer summarises the normal lesson something like this: You often start together to discuss and sum up issues you have been working on. Here you try to get the students ahead and maybe you give some recommendations to their work with the booklet and then they work together two by two. – After this summary, Arne wishes to add something:

> Arne: No, but clearly sometimes if they're in the middle of a process where … I know they need more time, then the lesson just starts with … : proceed from where we left off yesterday. That is, if I've seen and registered that they need another lesson to do this, or a somewhat more comprehensive problem, then it's just a matter of (…) yes. Getting on with it. (l. 175-178)

Asked what characerises a particularly good mathemtaics lesson Arne says: ”The students take over … one hundred per cent”. He uses an example from the KappAbel project to elaborate on the point:

> Arne: They took one hundred per cent charge of their own learning. Well, they came up with an idea, and they took it all the way through. And that goes for ordinary lessons too. …. / Well, as long as it is related to the subject or like …/ I suppose it isn't always all that much about [the subject], but if I think there is a potential for development in it. In terms of mathematics, for them, then I just get them going. (…) And then I often find that when they've been at it for a while, it's difficult for me … to immediately grasp what they've been up to, so then I often have to spend <u>a whole lot of</u> time on it, a colossal amount of time on trying to understand how they've been thinking. This is <u>one</u> type of quality about teaching that I really appreciate <u>enormously</u>. (l. 239-250)

This indicates a view of the students as active learners and a view of himself as a teacher, who needs to respond flexibly to where the students' activity takes them. To Arne, the KappAbel competition appears to a good setting for him to realise that view..

> Arne: Another example is this where we should work on Pythagoras. It was last winter too. From KappAbel some years ago, we used a puzzle.[7] We had som pieces

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[7] This problem (no 6, semi-final 2001) is not typical of KappAbel. At least not the way it is used by Arne in the classroom where we suspect a ”guided discovery” thinking and where the "product" Pythagora's theorem is most important. In most KappAbel problems it is not the product but the process which is most important. If you read problem no 6 in the context of the competition it is also the process perspective which is the managing principle. The question regarding the figure sounds like this: "This puzzle might be used for proving a well known geometrical theorem. Which one?" (Holden, 2003:116). It is assumed that the students know Pythagoras. They receive an envelope

> from a puzzle and had at first to do a big/ A small square. Or two small where the one was a little bigger than the other. And then – on this basis they schould make a big one with the same pieces. And the students spent an hour on this. A lesson, and at last I had to give them a hint. And this was intense mathematical activity engaging the whole [class]. Absolutely everybody was active. /.../ And then I used three minutes to explain Pythagoras. After they had been doing this activity. And ... I think that [they] understood it much better, than if I had used another approach. (l. 251-268)

When the interviewer asks, if KappAbel has been compatible with the other mathematical activities in which he wants to engage the students, Arne answers briefly: “I guess this is one of the reasons why I participate in KappAbel, that is the kind of problems that I,… I don’t mean qualification, but they are compatible with what I stand for in a way …pedagogically. It is the kind of problems that engage all the class. (l. 390-392)

From the student report, we know that their deliberations about the project began as early as between the first and second rounds of KappAbel, and intermittently, they spent 5-10 minutes on discussion. It started out with a common ”brainstorming”, where Arne encouraged them to share any thoughts they might have:

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with five pieces (rectangles and triangles) and the acticity is more over in the genre of Tangram.

> Nobody was allowed to say that an idea was bad: It could well be that someone else might see possibilities in the idea. We had to delay the discussion until we were certain that everyone had mentioned all of his or her ideas. (Report, p. 1).

After having discussed a number of problem definitions they chose to explore ”whether human beings are put together haphazardly, or whether there are connections in the body in which there is a mathematical system” (Report, p. 4). The project was interdisciplinary from the outset, and they wrote down all the rules and connections they could recall from the Arts and Crafts lessons. The golden section was one of these connections. They spent a Religion lesson discussing plastic surgery and ethics. ”There were many different opinions. Mathematics is clearly not value-neutral!” (Report p. 2).

Throughout the project period, some of the students worked on a large, complicated construction for which they used a computer programme. ”When they saw the final result they discovered that [it] wasn’t altogether accurate. They saw this during the process as well, but chose to continue. All the same, they were very pleased” (Report, p. 4). – Like Arne formulates it: the students take over, also in their conluding remarks:

> Our problem definition was about whether human beings are put together haphazardly, which they are not. It makes us speechless to see all the connections. Maybe something to consider in the debate on whether human beings are created, or are the result of an evolution taking place over millions of years? (Report, p. 10).

Matematics learning is happening when the students take over

Arne’s intension is that in his and his students’ mathematics classroom, student activity comes first and his own explanations come second. If he thinks that there is a learning potential in some students idea and as long as it is related to the subject of mathematics, Arne “just get[s] them going”.

Arne is educated in pedagogical and mathematical environments where the current reform of school mathematics is very influential. The priorities of these environments are highly compatible with those of KappAbel. The role of the competition as a possible external source of influence is therefore different in the case of Arne, than in most other situations. In this case KappAbel may not influence beliefs and attitudes by introducing novel modes of participation in mathematics classroom practices. But one may speculate that they support existing practices but supplying valuable tasks and problems for the students to engage in. In this sense, KappAbel may support and further develop the views and practices that already dominate

Ola and his students

Ola is the teacher in his 40s with 10 years of teaching experience in mathematics whom we first met in chapter 5.4. He has 10 Credits in mathematics and does not consider himself a mathematician. Through Ola's answers in TS2 we get the impression that in his school mathematical priorities he puts equal emphasis on the four options: use of mathematics, creativity and investigation, logic and reasoning and basic skills. Good mathematics teaching is to him characterised by discussion, co-operation and investigative activity. It seems like co-operation is a central dimension in his understanding of how to learn mathematics. In this section, we meet Ola again together with four of his students, Fredrik, Håvard, Mari and Thea and the 9th grade project work on ”Mathematics and the Human Body”.

Mathematics is chapters in the textbook

In the interview Ola presents his role in the classroom as ”the boss”: he who is in charge professionally without being authoritarian. First of all he characterises himself as a real ”text-book teacher”, ostensibly because:

> One of the reasons that one has turned into a real text-book teacher is that we[8] haven't managed to gain support for obtaining a lot of, well, materials for concretisation (manipulatives). More alternative teaching materials for maths – we haven't gained any support for that. And we haven't managed to make time for producing any ourselves, you know, and then there is nothing else, then all we have are the textbooks. (l. 121-126)

The students' corresponding first reactions as to what characterises a ”really good mathematics lesson” are the following:

> Håvard: Maybe the first lesson we have when starting a new chapter. Because then ... the teacher shows us what to do, after all. And after a while it's just / then we just continue the work on our own, and we put our hands up if we need any help. But it's like first they show us the first things we need to know from the chapter. I suppose that's what's / well, I don't know exactly
>
> Thea: ... I think it's getting a <u>really</u> good introduction to what we're supposed to be doing.
>
> I: How about you then, Mari?

[8] Ola uses this ”we” several times in the course of the interview, probably about the group of mathematics teachers at his school.

Mari: Well, I suppose it would be the same as / The first lesson is rather important when you start a new chapter, really. So that maths lesson one XXX learns, maybe the maths lesson during which one learns the most in the course of ... an entire chapter.

I: Could you describe a little bit about how you start a lesson like that? Or ... do a lesson like that?

Mari: ... I suppose we start out by doing the activity listed right after the beginning of the chapter. And then we get our rule books out while the teacher writes down some of the first rules from the chapter on the blackboard, and then we try a few exercises.

I: You think that's a really good lesson?

Mari: Yeah, reasonably good at least.

I: Okay. And how about you, Fredrik?

Fredrik: Well, it'll be much the same as what those guys are saying; we learn something new in every lesson. So that we have xforx the tests.

I: How about you?

Thea: Well, it's the same because we get such a really good introduction that we actually learn something new, so we're not just working on the week's work schedule, but we're starting something new. (l. 30-64)

Their KappAbel Report contains the first list of what they name ”problems” prepared by the class for the project. Here we also find the two headings ”Calculus of Probability” and ”Statistics” from the textbook – together with topics such as ”Relationship between height/weight – historically” and “Shoe size in relation to height – for each gender”. As with many of the other classes who went through to the final, their class ended up choosing the golden section as a central aspect of their project. Their inspiration came from a subject other than mathematics, namely the arts lessons, where they had recently been dealing with the golden section – symmetry, proportions, and drawing the human body.

You learn mathematics when you co-operate

Ola and his students agree that co-operation is important in mathematics, but the students do not agree with Ola when he presents co-operation in groups as the most important method of work, as something they normally practice once or twice a week in the classroom. They obviously understand "co-operation" in different ways. To the students, "co-operation" and "working in groups" is when they work together on a common product (problem solving, project) like for example the collaboration in KappAbel. To Ola, "co-operation" is also when the students sit together in groups, working each on their work schedule and helping each other.

Ola has found a role model in the mathematics teachers from his own school days. He liked very much the subject of mathematics and used to assist the teacher:

> I was used myself as a sort of helper, how should I put it, a sort of auxiliary teacher. And I thought it was great fun, and I think I learned a good deal of mathematics from XXX. So there was probably a … something to be gained from the good model one had, especially one teacher I had. So there's that, and then there's something about what sort of values you feel that you should have as a teacher, and what you should try to get across. (l. 397-401)

One of his students, Håvard, describes the normal mathematics lesson in the following way:

> We continue our work with the week's work schedule from where we are. Then we're just working, starting with the exercise we're at, and then if we need help with something we ask the teacher. We just work completely on our own. Well, we can always co-operate with others, but we just go on working to get the tasks on the week's work schedule finished. (l. 69-73)

Here the Interviewer asks whether the students experienced the work they did during KappAbel as very different from the ways they normally work:

> Håvard: Yeah. We don't, sort of work in groups normally, and all that.
>
> Marit: In any case very rarely.
>
> Thea: Yeah, and there was more practical work now, like making models in the right proportions too.
>
> Håvard: Looking for things on the internet and facts and lots of typing on the computer. We usually write and look for things on the internet, but not during maths and all that. So that was like (...) a little different from the usual routines.
>
> I: Yes. Did you experience that too, Fredrik?

> Fredrik: Well, it was good to have a bit of a change from the usual routines because ... we usually just do exercises and that sort of stuff. But now we got to work with Excel and learn a bit of that and got some other maths tasks to solve. (l. 291-310)

It was the teachers, Ola and an auxiliary, who picked the four finalists. But prior to the selection, the whole class had listed criteria for who should be on the team, with co-operative skills topping the list as the first criterion:

1. Ability to co-operate well with others.
2. Must be a good talker, and must know what they are talking about.
3. Be good at maths.
4. Creative.
5. Able to take pressure!
6. Take responsibility. (Report, p. 3)

The value ”co-operation”, which seems to be quite central in Ola's conception of his own teaching as in the KappAbel competition, is on top of the class list, whereas being good at mathematics only comes in third. We also note that "taking responsibility" is at the bottom of the list.

According to Ola, the good mathematics lesson is also well prepared. This means energy and high spirits which are important to learning, particularly for those who have learning problems. The same goes for the practical procedure:

> And the topic we had before that was geometry. Well, that's an easy subject to do with the students. After all, it's almost self-motivating because of the constructions and figures – it is one of the most concrete topics we have. And here we are lucky enough to actually have a few things at our school, which we lack otherwise, like being able to make it a bit practical. Especially with a view to the students who are <u>not</u> very good at the subject it's / … there's a bit of practical work and conversation and some activity, and then one is able to reach a few more of them. Then there's the – the approach I take is to try to catch the attention of the students who are not particularly interested in mathematics. Making sure it is not at the expense of those who want something. The ones who are <u>here</u> – they're, after all, the types who are interested in mathematics. So that interaction there – that is difficult because that thing about weak students does demand a great amount of energy. (l. 65-75)

KappAbel

KappAbel ended up being an extraordinary experience to both teacher and students in this 9th grade. Initially, Ola took charge when the class joined KappAbel first round. This is Hårvard's description:

> We were told the day before that we would have a two-hour test the next day, and not everybody was particularly happy, but the tasks were fun. It wasn't the way we had anticipated – that it would be just maths sort of, dead boring arithmetic tasks, but there was more problem solving and I think that is a lot more fun than usual. But we hadn't prepared much. (l. 148-152)

Both Ola and the students note that the KappAbel project has taken an awful lot of time. They even experienced a great strain in the project work, because they did not receive the message of being in the semi-final until 10 days later than the other teams. This was due to technical problems with Ola's computer. The students took over responsibility and worked hard three groups: the calculation group, the measuring group and the modelling group, while Ola acted as technical adviser and practical assistant. In the project report, the class tells that only a few students knew about the golden section and phi before the project work, and that their attention on the deliberate use of the golden section had been enhanced. In their work they had given priority to the practical approach and to mathematics as tool in investigating the human body.

Ola thinks that the inspiration to his way of teaching has come from what he has studied, experienced and seen. His teaching practice has developed on the way. When he finds something that works, he sticks with it. But:

> I don't think I've become static, I don't think so because you have to change from … class to class, don't you? And maybe you have to change from … one lesson to another lesson because your class is in a different way, on a different planet – it may be something that happened during the break, or they're not as concentrated, or maybe it's late in the day, or it could be right after a holiday or something like that. (...) It's something about sensing where I am and where they are, I think. (l. 406-411)

Like this Ola indicates that he is not only engaging in different mathematics classroom practices in different classes, but also that practice changes from one lesson to another in the same class. As we have seen above, the students perceive of the activities in the competition as totally different from the usual teaching. Thea mentions the difference between the usual tasks and KappAbel:

> (...) Those tasks in the book are really quite different from the KappAbel tasks. It's more like ... mathematical – the stuff <u>we</u> are doing at school instead of that problem solving kind of thing. You don't, sort of, have to think as much. (l. 117-120)

It appears indirectly that her perception of mathematics is confined to solving tasks by use of well known methods/algorithms, and that mathematics in school is routine, whereas the reflection and creativity needed in KappAbel is not part of this. So here we find a great potential of changing views.

From the interview, it appears that the students think that what they were doing in the competition was different, not whether they found it fun to work with mathematics in a different way. What they relate is mostly the pressure and the success. Otherwise they emphasize the fun caused by the circumstances – the fuss about their class, the prize, being the best in the county etc. To Ola, the teacher, KappAbel is a nice offer to the mathematics room but only as a break in between the usual activities – like some kind of excursion. As he puts it: "So … I can't boast that KappAbel has become an integrated part of our activity."

Finally, there was the student Håvard, who towards the end of the interview made the following point on KappAbel:

> I think it's sort of fun, there's been so much media attention about young people being so <u>poor</u> these days at maths and reading and all that stuff. I think that everybody who's down here, all the classes who are down here too, have shown the opposite; because I think everybody has done an incredibly good job. … Good projects and good presentations, they've <u>succeeded</u> at this – I think it's great to see that. And I think it shows that young people <u>aren't</u> that slow. (...) I think that it's really good that they manage to show that young people aren't as slow as they're made out to be. (l. 537-543)

Peder and his students

Peder is the teacher in his 60s with 39 years of teaching experience in mathematics whom we met in chapter 5.3. He has more than 20 Credits in mathematics and he is undecided as to consider himself a mathematician. Through Peder's answers in TS2 we have the impression that his school mathematical top priorities are that students learn how to master basic skills, and that the instruction is organised with a view to let the students attain routine proficiency in the subject's methods and skills. According to the questionnaire the students often work alone, but co-operation (mostly in pairs) is also common. He considers instruction as an interaction between student activity and the teacher's transmission of knowledge. In this chapter we meet Peder again with five of his students, Kristian, Per, Tina, Tonje and Renate and his 9th grade project on "Mathematics and the Body". When the students were interviewed – at the beginning of the academic year 2005/06, they had a new mathematics teacher. In the presentation we also refer to a single visit in the class during the project period.

Mathematics is a landscape and the students are guided by the competent teacher

Peder was asked to describe a normal mathematics lesson, at the very start of the interview:

> P: Well yes. At the moment we are working on a chapter that is about … the volume and area of different kinds of three-dimensional shapes. That is, prisms and cylinders – and we've been working with those for a fortnight now and are about to switch now and work with pyramids and cones, where we will then, of course, be doing practical exercises with these units. Surface area and volume area units and conversion of these to suitable dimensions for each purpose. And then we've been doing a good deal of practical work with these things. Like they've measured a few figures, and calculated the volume. And … and then in connection with KappAbel we had this box we were supposed to make for sending that figure, which was a correspondingly practical task where they had to estimate what size the different pieces had to be in order to make a box that fits that figure.
>
> I: What other practical measuring /
>
> P: Well, it's … / Things they're asked to bring from home, or to do at home, right. For example toilet roll or one of those kitchen roll holders and a milk carton and juice carton, and the classroom too, we've measured and calculated. (l. 35-50)

This is what Peder tells about the typical procedure of a lesson:

> The most common thing in one of my mathematics lessons these days is that it starts off with a short introduction to the topic that is to be worked with, but since we have period plans spanning, often spanning two weeks, not all mathematics lessons end up starting like that. (…) (l. 86-88)

When the Interviewer asks the students to think back to the time when Peder was their teacher, and then to describe a normal mathematics lesson, their story corresponds to his:

> Tonje: First we had maybe half an hour's teaching, and then fifteen minutes of work.

> /.../
>
> Kristian: And if you were wondering about something, then asked Peder, he would show you an example of something, then he'd help you solve the task, so if we made any mistakes, then he knew what we were doing wrong, and/ Yes, he was very good that way, showing examples.
>
> Several Students: Mmm.
>
> Marianne: He was very good at explaining.
>
> Per: Mmm.
>
> Marianne: Not all maths teachers are equally good at explaining, sort of. They may be dead good maths people, but like they in a way don't have the capacity to get it across in the same way. – So I think that was good about Peder.
>
> Tina: I think so too. We understood. If we were stuck, we understood after he had explained it. In this / Then we could solve all the similar tasks in a way, at least I could. (l. 143-176)

The class is gathered for presentation of the topic. This is what they see as "instruction". Then the students work on according to their schedule with tasks within the same topic. In the week of our visit, the headline of the math plan was "Chap. 7. We calculate with money" and the tasks which were to be solved during this week were set up in a schedule with three different levels. Peder tells that they nearly always use tasks from the book and sometimes a so-called "mathematical nut". The students tell that Peder often brought things to illustrate and explain, e.g. the oblique coin where the chance of heads is twice the chance of tails, or the yellow dice which they threw 100 times. Kristian says that he sometimes put the book aside and brought in Sudoku for example. This promps the following remark:

> Tina: Yes, breaks. Relevant to maths and somehow helping to develop our logical sense, and whatever. So it had to do with maths, but it was in a way a break from the book. This also in a way may still make us find that working with maths is fun. (l. 293-295)

Compared to their usual teaching, they experienced the KappAbel problems as something completely different. Here it was not a matter of using formulas or methods they had just learned:

> Tonje: We sort of didn't need to use the kind of formulas we've learned. It was mainly about logical /
>
> Tina: And what you can think of applying within the same theme, sort of.

> Tonje: Yes. Lots more problem solving, that sort of thing. We didn't need to know how to do equations to solve those tasks, for example.
>
> Kristian: No, [in] the final – then you couldn't <u>calculate</u> the answers. (l. 410-417)

Neither did the work with the project resemble anything they had ever done before in mathematics. From the class process log we learn that they have formed a number of groups – among these an organisation group, which in reality functioned as a management group: ”The organisation group informed the rest of the class of the plan for the next few weeks” (1st March, Process Log). According to the log, all decisions were made democratically, through voting and majority outcomes. Peder tells the team who was sent to the final was appointed by the class through a secret ballot which showed one hundred percent agreement, and the very same five students he would have picked himself were chosen. One student said: ”We all voted for the same ones. One boy – who gets A's [the best mark] – and his best friend. He crams, but he's not as good. Three girls are going, in order to have a stand-in in case someone falls ill.”

In this winner class, a project was also developed on the basis of Da Vinci's ideal man and the golden section, and the students collected data by measuring each other. According to the process log, we[9] – ”two ladies from the Norwegian Centre for Mathematics Education” – visited the class one week after they had started their work, when a new group was formed to design a box to hold a figure. The exhibition group was assembling this figure. The measurements of hands and fingers had proved too inaccurate, so they had decided to measure again, using a slide gauge in order to be able to include an extra decimal. The poster group started planning, and the organisation group drafted texts for the introduction and for the problem formulation.

During our visit, we sometimes heard people around the room saying: ”But the Da Vinci-character is a male adult.” When we asked what they had chosen as their problem, one of the students responded: ”The Da Vinci model – does it apply to us? Who did he model? Perhaps a man from a different ethnic group?” In the report they write that they want to find out if Da Vinci's theory applied to themselves, if the proportions of their bodies were consistent with his theory of the ideal man. Hence they measure and calculate the relation between their height and arm's length for example.

It does not appear in the log what mathematical discussions the organising team had on the basis of the tables generated from the new data of the class (two

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[9] Tine Wedege and Kjersti Wæge.

boys had keyed the results to two decimals in an excel spreadsheet), but we were present during the discussion of the group. Here, one of the diagrams they are studying shows the relation between the height and the arm's length of the girls and Tina has a short debate with Tonje, who functions as the group's secretary:

Tina: We need to agree on this before we go on.

Tonje: But, we are supposed to have a finished product by Friday.

Tina: OK, then we'll delete the line in this chart.

They were here talking about a line that was supposed to show the linear relation between height and arm's length of the girls. We see how the product orientation and the time pressure function is putting an end to the discussion, and – in the report – they did not try to approximate straight lines as graphs of the proportions of the ideal man. Neither the relation between the boys height and thumbs breadth, which in the working phase acutally showed a linear relation.

From Peder's account, we know that he did not act as a mathematics teacher during the KappAbel project, but more as a practical adviser. That was also what we noted in the observation situation where the organising team which was identical with the semi-final team, Kristian, Per, Tina Tonje and Renate, were discussing tables and diagrams. Exactly at this moment, as described above, Peder entered the room and stayed for about two minutes. He simply asked: ”Are you deciding whether to include everything [all the charts]? Things happen along the way, after all, and you're going to have to make new decisions.” Peder did not engage in the interesting mathematical discussion about units in the systems of co-ordinates or the accuracies that the diagrams and their observations might have resulted in.

Learning mathematics is the teacher presenting and students solving tasks

In the interview, which was made not until the following school year, the five students apparently have not changed their perception of mathematics and how to learn it. When the interviewer approached the question of what they consider the characteristics of a good mathematics lesson, this is how they responded:

Tina: When we learn something maybe?

Kristian: At school?

I: At school, yes.

Kristian: That we don't just sit there and work and that sort of thing. That we get a bit sort of xxx

Renate: Now it's teaching.

Per: The teacher does some teaching on the stuff we're supposed to learn first, and then we work on it.

Tonje: And then it's good if the teachers shows it to us with figures and stuff, because that makes it easier to understand. If not everybody understands, sort of, the theory behind it, it's easier if you get to see it put into practice in a way, I think.

Per: Yes, xxx if all they say is that "x plus two equals three", and don't explain why. Then you understand the point about it.

Kristian: Show it, then you see it /

Several Students: (laughter)

Kristian: I think.

I: And Tonje, when you said practice, what did you mean by that?

Tonje: Well, it could be figures, for example. For example those, what are they called, tangents / no something or other like that.

Tina: Yes for example. That is something you can work on, if you're doing geometry, for example, or that building stuff, triangles, squares. And it could be / Eh, anything else it could be? (…) This is difficult, really.

/…/

Marianne: Eh, no that's what they're saying, right. That we're learning something. Not that we're just sitting there working, but that we're getting a bit of teaching, and that we understand what's behind it.

Renate: That the teacher takes the time to explain if there's something we don't understand. Not just saying that this is the way it is, but also explaining why that is.

Per: And showing us why it is like that. (l. 36-123)

You learn mathematics when the teacher goes through the subject preferably with figures and good illustrations and then afterwards the students work on the subject solving tasks/problems. Renate and Per say that they want a conceptual knowledge, not just an instrumental one. They all tell enthusiastically about a lesson where Peder had brought containers which they filled with water in order to learn about volume.

KappAbel

It is the KappAbel semi-final in Arendal that made the greatest impression on the students. They say that, at the beginning, they had little idea of just how big the competition was.

> Marianne: It was great fun
>
> Tina: The focus was on maths, sort of, but in a <u>different</u> way.
>
> Tonje: And then it was very social too /
>
> I: Elaborate on <u>that</u> a bit please, Tonje.
>
> Tonje: Well, it was a bit sort of. xxx The tasks/ they were <u>fun</u>. It wasn't like I was feeling all the time that ”Oh God, it's a competition, we must do well now, I must deliver” and that sort of thing, but it was a bit more sort of, more sort of /
>
> Per: It was <u>fun</u> to take part.
>
> Tina: It was <u>fun</u>, it was a <u>fun</u> competition. More sort of friendly and “Oh well, it doesn't really matter if you don't do all that well, you've done your best”, and sort of like “Now, got this far, good”. (l. 791-806)

When asked about KappAbel's significance they return to the topic of mathematics after having mentioned the prize and the reward:

> Per: It's been a fun sort of <u>maths</u> experience, it xxx meant to me, it has / I don't know exactly what to say about that. Above all that it's been fun.
>
> I: Mmm, the <u>social</u> bit too, right?
>
> Per: Yes, all of xxx, and then it's given me a new / what should I say
>
> Tina: Opinion
>
> Per: <u>Opinion</u> about mathematics
>
> Kristian: <u>View</u> on maths
>
> Several Students: (laughter).
>
> Tina: But I think I've sort of learned new ways of <u>thinking</u>. Because I haven't sort of been thinking sort of in terms of <u>ways</u> of, sort of, solving tasks, like I was <u>doing</u> then. [faulty recording] xxx in that sense, maybe / (laughter). Become more like, logical.
>
> Tonje: And then before it was like, especially when we were reading xxx tasks, I was a bit more sort of like ”nooo, I'll never manage this”, and then I'd give up. But at Kapp Abel we <u>had</u> to, since it was a competition after all, so then we had to xxx no matter what. So that's what I believe xxx I try a bit harder before I give up (laughter).

Per: Yes, making mistakes, and then looking up the key (laughter).

Several Students: (laughter).

Kristian: But there was no key after all, now it was just a matter of if you make a mistake, you've got to start all over again from scratch. (l. 955-082)

Peder relates his own pleasure at the kind of tasks he encountered in KappAbel:

Peder: Yes, I've always been / Found it fun for one to solve that sort of problem task. I've got lots of those sorts of books at home with the Problem solver one, two and three. And I've also tried to pass that down to my students to a great extent. … At the end of term and that sort of thing we often have brainteasers of that type.

I: Why do you think that's a good idea?

Peder: Well, I happen to think … / Instead of just … coughing up what the teacher says or what the textbook says about do this and that, they have to do a bit of independent thinking. They have to enter into the problem and produce a solution to it themselves then. Like there are no clear recipes to the KappAbel tasks, after all. We've also participated in these competitions called the Advent Calendar. I don't know if you know those? (l. 447-461)

He sees evident learning potential in the mathematical problems, but he has not integrated them in his school mathematical priorities.

But, did the four students perceive of the KappAbel problems as mathematics? And are their views of what constitutes maths widened – see the following comments prompted by the Interviewer's direct question, - a question that was actually not included in the interview guide:

I: Yes, in terms of perception, in terms of how you perceive mathematics, or how / yes, anything

Marianne: I think it's positive in terms of how we perceive of mathematics now.

Tonje: No, I have a poor perception of mathematics now X I don't think mathematics is any fun now X.

Tina: But precisely that/ I thought mathematics was more fun, xxx sort of not seen that side of mathematics all that much, before that is. So I thought it was fun to solve that type of task.

Per: Instead of just sitting there with a pencil and a book.

Tina: Yes. But that does not mean that I think the other maths is that much more fun, because it's in a way back to what used to be. So in a way xxx back again. But like, if I was going to work with maths, I'd do further work with that, more those types of field, I think. I found that xxx fun xxx.

Marianne: Xxx sort of <u>seen</u> that there are other things to maths than just the <u>boring</u> ordinary like xxx before.

Tina: I'd never <u>heard</u> about phi and that sort of stuff before.

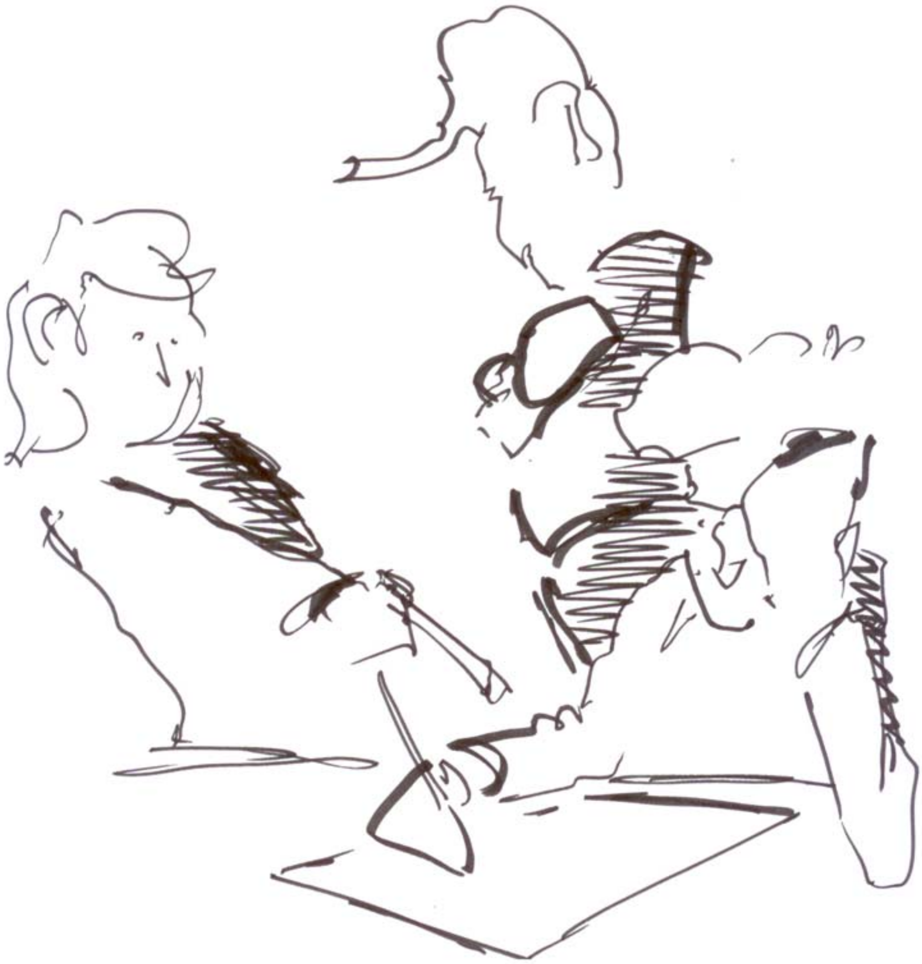
Tonje: Phi?

Tina: Okay, right, X you haven't heard about it either X

Tonje: (laughter)

Tina: Some people gave a presentation on phi. (l. 829-854)

If, like in this case, the KappAbel competition is isolated and is followed by usual teaching, the students get the idea that creative mathematics is the exception of the rule and their motivation might fall. This class has experienced something different with mathematics but – in the following school year – they had another teacher a mathematics teacher with whom they have to establish a new didactical contract.



Randi and her students

Randi is the teacher in her 40s with 12 years of teaching experience in mathematics whom we met in chapter 5.3. She has high credits in science but not in mathematics (5 Credits), and she does not consider herself a mathematician. Through Randi's answers in TS2 we get the impression that in her school mathematical priorities she puts equal emphasis on the learning of everyday mathematics, creative and investigative mathematics, logical reasoning and interconnectedness of mathematics. At the same time she values highly the mathematics teaching where students acquire safe routines in the methods and skills of the subject. In this section we meet Randi again and we are introduced to his students' views of mathematics, as they are expressed in the project of "Mathematics and the Human Body".

Mathematics is fun – or should be

When the interviewer asks where Randi found inspiration to her way of teaching, she tells that she does not remember her old teachers' mathematics lessons – only if they were good or bad – and she continues:

> R: So it's ... more through experience. Maybe that's the kind of person I am: when you have been teaching for a while, you find out what's working and what isn't. If I have used the parts of my personality that work, .. it's instinctively, that would not be a conscious choice. But the thing about the lessons being like a game, well, I think it is important to make the students think it's fun, that's … I've probably found inspiration here and there. Among other things/
>
> I was at another school with another teacher, a much older physics teacher who played his way through the physics curriculum with trucks and balls and stuff. You take over things like that. How to make mathematics fun? If he can make physics fun, which is actually hard to do? So how can you manage to make the other subjects fun? (l. 306-316)

When Randi is invited to say what she has most at heart at the end of the interview, she states:

> Well, the important thing is just to go way back and remember how it was to be thirteen, fourteen, fifteen, sixteen years old. That's the most important, I think. Try to make mathematics learning a game so it isn't just dry and dull, just the same exercise over and over again in order to practice at tests and examination, and forget the whole thing after half a year or three years. That's the most important, I think … That the students think it is fun. So when they grow up, none of them will say: Oh, I

just had a mental block when I saw a math task (laughter). Mm. So that's the important thing. That they should be fond of mathematics. (l. 590-598)

Learning mathematics is to communicate

From the questionnaire we had the impression that spoken communication and discussions about mathematics constitute a significant dimension in Randi's approach. During the interview she elaborates on this:

> R: You can talk about mathematics. So, if I have one way … of calculating percentages, for instance, and another in the class has another way of learning to calculate percentages and a third maybe (laughter) has learnt to calculate percentages in a third way. So we can start by asking … what is actually right or wrong. Or … if some one in the class says, … it doesn't matter what way you do it as long at you get the right answer. And then it's OK/
>
> Sometimes we just go off at a tangent, then we may suddenly talk about calculation of per mille or calculation of currency or the like. Because you use the same kind of formulas. So that's the kind of discussions you may have. (l. 266-274)

According to the Log Report, Randi's grade 9 discussed in the whole class and later in two groups. They chose to split up and work on two different issues within the theme of ”Mathematics and the Human Body". One of the groups spent "a whole hour" to formulate their problem because they had a series of mathematical concepts involved: phi, Fibonacci, the golden section etc.

KappAbel

Randi finds that the problem solving in KappAbel is compatible with the usual activities in her mathematical classroom; but not the KappAbel project work:

> Well, spending time on project work that is not, doesn't correspond with my intentions in mathematics teaching. My students should be allowed to wonder, yes. And they should get answers to their questions, yes. But making such a big work out of it, I just don't … And then again KappAbel has an exposition, of course there is a lot of mathematics in arts and crafts, but I don't spend time on it in mathematics except from talking about it and discussing it and so on and so forth. (l. 405-411)

The Interviewer asks her to elaborate on the fact that she wants them to wonder; and Randi continues:

> I don't take them out, but … the important thing is not where we are actually, but statistics for instance, probability, it's so easy when they have a question not just to give the answer in the classroom and show it on the blackboard, but to go out. Sit

> down on a bench, watch forty people passing by, how many are wearing this or that article of clothing for instance. This experience. This kind of everyday things. I really want them to take it in, not to be something distant, mathematical, with formulas, and then you just type it in at your PC and something comes out. I so want them to take it in. (l. 425-430)

During the project period with empirical investigations, they discovered that it was possible to connect their two problems by media of the “golden section”. They were very pleased “this is fun”, but they also found it a little “sinister”. The class has been working “over time” and they are pleased with their result. And they also conclude “We have had many new experiences during this work. We have experienced that it isn’t always that easy when the whole class has to co-operate on a project.” (Report, p. 3)

Without the students’ voices from an interview, it is purely guessing to conclude anything on the potential of KappAbel to influence the views on mathematics and teaching/learning practices of Randi and her students. Problem solving and project work in the KappAbel sense do not appear to be a part of Randi’s school mathematical priorities. However, there seems to be a potential of changing practice because the views of mathematics in this classroom are compatible with those of KappAbel.

Steinar and his students

Steinar is the male teacher in his 30s with five years of teaching experience in mathematics whom we met in chapter 5.4. He has a strong background in mathematics (20 Credits) and considers himself a mathematician. Through Steinar’s answers to TS2 we get the impression that he emphasises the students’ learning of mathematics that they will need in their everyday life. When he is asked to characterise good mathematics teaching, he allocates almost the same points to discussion and co-operation, to routines and to project work – yet the least to the last activity. In this section, we will meet Steinar again together with four of his students, Anders, Stian, May and Toril[10] and the 9th grade project “Mathematics and the Human Body”.

[10] Unfortunately the first part of the interviews was ruined. From this part, we only have a few notes.

Mathematics is a tool made of rules

The interviewer asks Steinar to describe a normal mathematics lesson. First he explains that they follow the chapters of the textbook and their content rather slavishly: ”We go through the constructions on the blackboard, they work with exercises, and I go round supervising them and … that sort of thing.” Then he gets more specific:

> Steinar: Yes. … We have three lessons a week. And about fifteen minutes of each lesson I go through what is to be done during the lesson, and what they should finish by the next lesson. … It's on a Monday so … I give them the entire week's work schedule that they can work on independently. So in a sense it is optional whether they want to listen to me going through the topics or work on their own. I think most of them listen to me. And then, what I go through, they write down in their rule book. … And then they work on exercises, they sit in pairs and work together. … The week's work schedule is differentiated into two sets of tasks because this group of mine is fairly homogeneous, we're divided into homogeneous groups. So I … have a good group, so I have the most difficult tasks. And then there's / Not everyone is equally good after all, so they get a sort of medium type of task and work through those. … Next lesson, I also go through a bit of what they ought to get through during that lesson, unless they are already at that point. And I do questions on the blackboard, if someone's wondering about some question or exercise, if many of them are wondering about the same exercise. But apart from that they do work very independently, they're a very independent group. They ask each other a lot, and are very easy to deal with (Laughter) (l. 45-60)

During the exposition, Steinar writes one or two rules on the black board; he goes through one or two examples and asks the students to write down the rule in their “rule book”. The tasks are from the book, but when the interviewer asks Steinar, whether he has any kind of dialogue with the student while going through the topics on the blackboard, he says:

> S: Oh yes, mm. I try to retrieve information they know from before, and … much of it was dealt with last year and the year previous to that, after all. Much of it is repetition so … it's a matter of getting a dialogue about what they remember from last year up to now, and what they can try to extract from … new things and try to see a connection … yes. Then we often have, or we sometimes, sometimes we have, well it's not in <u>every</u> lesson, but then we do have a bit of those kind of mathematical brainteasers, where we talk a bit about maths. A bit of that problem solving sort of thing, we have a few of those spicy tasks now and again. … They're similar to the ones in that Advent Calendar, if you're familiar with that, with … “matematikk.org” … do you know what I mean?
>
> I: Yes
>
> S: Yes, those types of tasks, and having a bit of talk about them. But forty-five minutes, they pass very quickly so … it's not always that we have time for all that much of that sort of thing. (l. 74-86)

Steinar uses ”brainteasers” and other problem solving tasks to “spice up” his teaching – something extra, not as part of what he perceives of as the normal teaching of mathematics. ”It gets to be a bit too much a matter of routine, I'm starting to think, but I suppose I'll learn a few new tricks as I go along” (l. 177-178). It sounds as if he considers development and change as something coming from outside in the form of tips and tricks. Actually, Steinar calls the KappAbel problems from the qualifying rounds and the class activity around them ”a nice interruption of the teaching”.

When the interviewer asked the students, how they experience the similarities between the way they have been working during KappAbel and what they normally do during mathematics lessons, they react as follows:

> May: it is maths after all
>
> Toril: equations and that sort of stuff
>
> Anders: We do have tasks sort of, some places. After all, yes, they're all tasks [exercises?], they're sort of solved in a slightly different way, for example. So in a sense each project is, after all, an exercise. You sort of solve it in a slightly different way. Well, you may have to work a bit more (laughs).
>
> I: Have you ever thought: But, this has nothing to do with mathematics?
>
> Toril: No, I haven't at least.
>
> Anders: No, not really. At least not about our […], to put it that way. But some of the others were a bit more difficult in terms of discovering the mathematical element. Or there was, there was mathematics there, but it was a matter of / (l. 276-293)

Steinar describes the class as quite homogeneous, and normally the students engage with standard tasks from the textbook listed in their working plan. Steinar encourages a couple of the students with more challenging tasks explicitly made for the students who do well. Most often they, however, do not take up the challenge.

From the Process Log, we get the impression that the students do not see mathematical competence and problem solving as the same: it is quite possible to be good in maths without being able to deal with problem solving and vice versa. The class agrees that the four members of the team going to the final ”had to be good at maths and able to do problem solving tasks” (Report, p. 5). The character of the KappAbel problems is such that there is not just a single mathematical rule to be followed in order to solve them.. According to what seems to be the didactical contract developed between Steinar and his students in this 9th grade, mathematics is a tool made of rules. Problem solving and project work do not appear to be included in the conception of mathematics in this classroom. As Steinar puts it: ” … it has been very busy, it’s been a somewhat unfamiliar … way of thinking about maths, bringing it in, integrating it into a topic.” (l. 239-240)

You learn mathematics using the rules to solve tasks

Explaining the teacher’s role in relation to the students’ learning process, Steinar says :

> Steinar: Yes, no, well, there are always questions popping up. So I’m going round supervising and listening to how they’re doing and: This is going well. And: You’ve grasped this point and that. And that sort of thing. Of course, I don’t do the problems (Laughter). … Well yes, I suppose I do have a dialogue going with them too … to get a few impressions of what they’ve understood. … Yes, I do have that.
>
> I: But when you’re saying that they work very independently, does that mean that they try on their own first, before they contact you?
>
> Steinar: Yes, they seem to try on their own first and then they mostly ask the person next to them, and then … if they don’t get an answer, then they ask me.
>
> I: Well, exciting, because now …
>
> Steinar: Yes, because I try to be around as little as possible, I want them to try to figure this out a bit by themselves. [inaudible] and then they’ll ask me for an answer and then I’ll say, “have you looked in your rule book?” - No, he hasn’t. “Do that.” Then they’ll look up their rule book, and for the most part they’ll find an answer. So I try to make them even more … independent, too. Yes, so I don’t always give them / I give them a few pointers now and again –I do that. (l. 121-139)”

And then they look in the rule book ”where they find an answer almost at once”, Steinar says. When the interviewer asks him where he gets the inspiration from for his teaching, he says:

> Steinar: … That may well be … what I think is the smartest thing to do, is to go through the topic and then for them to work with exercises. Because it’s … it’s the practise with exercises that makes, that I believe makes, them good at it after all… together with a bit of dialogue. … In terms of performance – and tests – you do, after all, depend a lot on a rule book, so I put great emphasis on good work with the rule book and that they’re able to find the right solution themselves by using it. … I’ve seen that’s important. A few of them are still struggling. I also teach tenth grade … not all of them are equally good at using the rule book, and they … well they struggle when we have tests even though they work well and – understand – during the lessons, it tends to disappear after a while so … I’ve come to understand that the rule book is … terribly important. … which is why I use the approach of going through topics in my lessons, and have them write things down. … Apart from that, I think it’s very important to differentiate the tasks they are given. … Previous classes I’ve

> had have had an even greater span, and then I've cut some of the syllabus … for the very weakest ones. … I've had four levels too once, where I've … left out a lot of the syllabus to return to more formal training and that sort of thing from time to time. So … classes are different, so I think differentiation of the tasks is very important. That they're given tasks they can cope with at the same time as the strongest ones are also given a challenge. (…) So I suppose it's more a matter of experience that I've found out these things. That I've seen that … the … weakest ones must have something different to do, otherwise they lose their motivation completely, or they just sit there doing just that – sitting (Laughter). (l. 187-204)

From his experience, Steinar knows that it is routine in solving tasks and use of rule books that make the students better in mathematics.

KappAbel

Steinar explains that it was extremely difficult to get the whole class interested in KappAbel. In the end only half of the students (10) participated in this part of the competition, while the rest had ordinary mathematics lessons.

In the Report, it is explicit that project work was a new experience – both to the teacher and the students. However, when the students got down to work they found it fun to work with the topic They had chosen an open problem on mathematical relations in the human body. In their work, they have only been looking for linear relations (direct proportionality) and they did not find any. However, they also regard this as a result.

Initially, the students did not perceive of the project work as a learning process. In their Process Log they write:

> In order for the project to be as efficient as possible, we decided that only half the class would be working on it. If the entire class were to work on the same thing, it would only result in chaos, according to many. In addition, not everyone was equally motivated for working on the project. (Report, p. 5)

This may be understood as if the students consider the product, the exhibition, the competition as the primary purpose of the project. However, from the interview and their report we also get the impression that they have both learnt from the project and enjoyed doing it. In the report they say: "All in all, we think projects in mathematics are something we ought to do more often. We think our chosen problem is original and slightly amusing." They elaborate on this in the interview:

> I: What has taking part meant to you?
>
> Toril: Meant?
>
> Anders: I think maybe I find maths more fun now (laughs)

Toril: Yes

May: Same here

Stian: Learnt a bit more

Anders: Thinking a bit, being allowed to think a bit differently about mathematics not just being about sitting there looking at your book and writing, sort of /

I: Yes

Anders: They're slightly different things and sort of ….working together and doing difficult things and trying to manage and cope with it all it's sort of completely different from sitting there reading from the book.

May: Yes (l. 206-226)

In this 9th grade, teacher and students want the mathematical classroom practices to be different. Steinar wants the teaching to be a little more "spicy" and he often thinks of using computers. It seems like the four students have broadened their ideas of what could be done in the mathematics lessons, but they are not sure if you really learn mathematics doing these kinds of activities (problem solving and project work). Their views of mathematics may not have changed much – problem solving still differs from mathematics, because it requires fantasy, creativity and energy. But in Anders' words, the students have been challenged to "think a bit differently about mathematics" when they did the project.

Øystein and his students

Øystein is the 30 year old male teacher with seven years of teaching experience in mathematics, whom we met in chapter 5.4. He has more than 20 Credits in mathematics and mathematics is the subject he likes the best. He considers himself a mathematician. Through his answers in TS2 we get the impression that he emphasises students' learning of the mathematics that they need in everyday life. He gives low priority to logics, routines and basic skills. Good mathematics teaching is primarily characterised by discussion, co-operation and investigative activities. From the questionnaire, we had the impression of co-operation for social reasons, but also as a source of mathematical learning. In this subsection, we will meet Øystein again together with four of his students, Jørgen, Espen, Katrine and Ingrid, who had done the 9th grade project on "Mathematics and the Human Body".

Mathematics is either technical or practical activity

Normal conflicts between ideal beliefs and everyday practices in Øystein's and his students' mathematics classroom emerge explicitly and implicitly in the two interviews. Øystein sees his own role in the teaching of mathematics as that of a guide or supervisor. As the one who tries to put students on the right track without giving away exactly where they are going, in order to generate the greatest possible activity on their part:

> So I try to be as small as I possibly can, when I have the opportunity, but obviously, when we go through things and summarise things, it's more like they need to get it down in their rule books, and ... It becomes more sort of disciplined, so to speak. But they are fairly free. They've got to take responsibility for this themselves,... (l. 271-274)

Apparently, his four students perceive of a mathematics lesson and doing mathematical exercises as two sides of the same coin; however, they also see concrete help from the teacher as a good thing. Their responses to the interviewer's very first question of ”what, in your view, characterises a <u>really</u> good lesson of mathematics?” sound thus:

> Katrine: … That the teacher helps you, that you don't sit there with your hand up all lesson, that you get tasks suitable to … what you can
>
> Espen: Your capacity
>
> Katrine: Yes, capacity
>
> (...)
>
> Jørgen: Yes, he has to explain well, so I don't just sit there understanding nothing of what he's saying.
>
> Katrine: Have a bit of … variation in the tasks so that it's not just the same old stuff with squared x and all that, that there are some nice tasks to work on.
>
> I: What are nice tasks?
>
> Katrine: Like the ones we've had in KappAbel, it's been nice to work with those tasks.
>
> (...)
>
> I: How about you then, what's a good mathematics lesson for you?
>
> Ingrid: … That I manage to do the tasks and that sort of thing
>
> Espen: Manage to master them.
>
> Ingrid: And understand them sort of after a while.

I: How often do you have a good mathematics lesson?

Espen: Almost every time. (l. 30-62)

The other pupils confirm Espen's assessment, and they also agree that they have a good and humorous teacher – which is something they attach great importance to. The students also find it necessary that instruction meets the demands of the individual students. Øystein agrees and makes the following elaboration:

> In my class it's like from the best to the weakest student in mathematics it's sort of, it's at least five years. And we have to ... differentiate. Our conscience won't allow us to do otherwise, but it *is* a bit difficult, because it takes up a lot of time. But that's the way it is, after all. That's the way it is for all teachers. (l. 144-47)

Already in this first sequence of interviews, we get an impression that the KappAbel problems are perceived as a welcome variation in the teaching.

Textbook and tasks have a central position in the students' consciousness. This emerges from their description of a normal mathematics lesson:

> Katrine: When we come in he says what topic, or what it is we'll be working on … we do go through chapters then the topic… And then … he explains a bit about the tasks we're supposed to start doing. For example if they're, like, graphs, he explains how we're supposed to do that, the different points to do with them, and then we do the accompanying tasks.
>
> Espen: Then we do tasks for a while and then we progress, then he explains a little more, together, and then we do tasks. (l. 78-84)

They report that about half the time is spent on solving tasks: first the teacher explains, and then they solve the tasks from the book afterwards. The tasks are adapted to the students, so that the ones with difficulties are given easier tasks. But when Øystein brings in other tasks than the ones in the textbook, the students are aware that something different is happening:

> Ingrid: He brings some in himself now and again
>
> Jørgen: Yes, all of a sudden he just thinks of something and writes it down on the blackboard and then we do them and then we get a bit of a challenge.
>
> Katrine: Or last year at least, then we got a few of those tasks from KappAbel that he'd taken from other years, when we were in Year 8. (l. 114-121)

Øystein reports that he gets tasks from the web or other textbooks, or that he makes them up himself. He has a kind of banks of ideas. The variation itself is important to the students. Thi becomes apparent, when they discuss the KappAbel tasks after the semi-final:

Jørgen: Well, I've started to get bored with those tasks, because we've done so many of those thinking task so then we're starting to get bored with them by now.

Katrine: … When we get … back to ordinary tasks I'm sure it'll be nice to have those again. (l. 217-220)

And somewhat later Espen spontaneously says: ”Now it'll be nice to get back to the old regime again.” (l. 275)

Øystein explains that he considers a practical approach to mathematics the ideal, and comments on his basic idea of teaching, and the role of the textbook in it by saying:

> That way of working is of course a bit cumbersome – because, after all, the textbook doesn't support it. In that sense it's an unsatisfactory tool, the textbook, in that sense. ... I long for a book that takes the practical aspect a bit more seriously. (l. 60-62)

But what exactly does Øystein mean when he says ”practical”? On the one hand, it covers topics which the students are interested in. He mentions a sequence about mobile phones as an example. But when he says “We take a practical approach to it” (l.164), it means that the students work on solving tasks that do not originate from the textbook, and where they do not necessarily make use of the textbook. The students appear to apply the same use of language in the following sequence, where they are talking about the preparations for rounds 1 and 2 of KappAbel, i.e. about problem solving but not about the project:

I: Were the preparations very different from the way you normally work?

Espen: Yes, I suppose it was more sort of practical, practical maths is KappAbel after all, it is *not* just sort of formulas and stuff, it is more probability and that sort of stuff I suppose.

I: How is it similar, or dissimilar, to the normal lesson?

Espen: Not very similar.

Jørgen: Spend a lot more time on the tasks and manage fewer tasks but you get to do a lot with one task, to put it that way. (l. 145-155)

In Øystein's example of an actual lesson which was very successful, he tells us about something he has tried before, for example with Year 8 classes: He asks the students how far it is from the school to the shop. The distance is about 700 metres, but the students' first guess varied from 130 metres to well over a kilometre. They were divided into groups in order to find the correct distance, and were allowed to use anything at hand that might help them solve the task. Afterwards, they were to present and discuss their approach:

> I don't think they'll forget that, I think, because they've done it. But if they'd calculated it, or just got it from the book, then we wouldn't get a practical approach. So I feel that it was sort a starting shot of … for that class to get started with, or practical things … for getting into place a few of those self-evident things that they need if they're gonna get anywhere, sort of, in lower secondary school. It is a … good example of the practical approach. Of doing things. Of something sticking. While just calculating may disappear just as quickly, although it took a lot longer. (…) So … yes. (l. 246-53)

Elaborating on the relationship between the theoretical and the practical he says:

> You have to put it into a practical context, (…) Those times when I've tried to teach them techniques before they've understood them – it hasn't worked at all well. So that's what we do. And especially all this about equations and that sort of thing. There we try to put it all into their own everyday lives so that they have something to relate to. It's sort of in line with this KappAbel stuff (…) We have one of these problem solving oriented approaches to it. Rather than one of those purely technical approaches.
>
> The point is then as they start seeing. As we take simple practical things to which the answer is very easily found, we try to put them into equations afterwards. So that they see how they can, or what it's good for really, or what the point of it is. (l. 40-49)

However, Øystein concludes his report by claiming that the "practical approach" is not always feasible: "But that sort of thing does take a lot of time. When you're pushed for time, you must necessarily take a more theoretical approach." (l. 292). Øystein, then, thinks of practical mathematics as in opposition to theoretical expositions followed by tasks. .

Learning mathematics trough practical co-operation

In the KappAbel project report, the students ask themselves in relation to any of their mathematical results: what are the practical consequences in or for everyday life? But what has participating in KA meant to the four students? What have they learned?

> Katrine: But we have <u>learned</u> something from it in any case, it's not that we've just done it to find out about lots of things – we've learned something from it too.
>
> (...)

I: What has taking part meant to you?

Jørgen: We've learned a lot of new things so it was nice to have learned it. [Is the transcription wrong here: Should it be "vært der" rather than "lært det" – in other words "been there" rather than "learned it"? – det kunne der måske være, men jeg tror det nu ikke…]

Espen: I for one have at least got much better at mental calculation because of it (Laughter)

Ingrid: And I have sort of begun to dare to talk to people more. Before, I didn't dare say anything to anyone, sort of, almost.

Katrine: ... I've understood the things a lot better when I've seen them in a practical solution, when I see how it can be applied, not just writing exercises. (l. 251-263)

Ingrids comment suggests that co-operation is an important part of thd students' experiences with KappAbel. Øystein addresses the same issue and elaborates on what he wrote in the questionnaire TS2, on co-operation:

> That's when I think it's interesting, when we get a clash, because then they sort of have to ... Then the groups have to split up a bit and start to discuss a bit, and the time passes and they get pushed for time and that kind of thing. And ... that is really where I see the usefulness of all this. The greatest gain is the fact that they can co-operate about a problem, and that they ... arrive at an answer nevertheless. (l. 379-83)

In all of the teacher interviews, the interviewer asked the following question: "This approach to teaching which you have developed gradually. How did you develop this approach?"

Øystein: I think that … this started with KappAbel some time about four years ago, or thereabouts. … It was an invitation. Where one saw really … that this thing about several roads leading to Rome is important, and that pupils differ. They think in different ways and they use different strategies. And it's sort of this that is the starting point. … Like for example these students that I have now, like there are two practically minded students and two more technically minded ones, that's what I think at least. (l. 301-308)

He elaborates a little later:

> Well, you see that it's a completely different type. But I think that this need has ... it has appeared because I see that traditionally ... it doesn't function sufficiently well. And as ... a teacher on my own with eighteen students, I for one cannot differentiate well enough in a traditional way, because then everyone would get different levels of theoretical tasks. While here they get different types of tasks that are

> approximately the same level, and that is a very different matter. (…) Yes. (l. 328-32)

From chapter 5.4, we know that Øystein thinks that gender matters in mathematics and points to the social advantage in having gender-mixed teams and groups. From the project report we have the impression that possible gender differences are also caused by the socio-cultural background of the students. On their findings with regards to the proportion between height and arm length, the students write, for example, that:

> The reason that men have longer arms than their height is perhaps that they work a lot more outside than the ladies do. It may also have to do with the fact that men are broader across the shoulders than women. Most of the people participating in this investigation were farmers. (Project report, p. 4)

Øystein concludes the interview by telling about a dream that he has on doing a pilot project à la KappAbel with his future 8th grade as a kind of experimental class. He wants to break away from the textbook as the dominant tool when teaching: Replace going through theory (textbook), presenting algoritms (textbook), and solving tasks (textbook) with organizing instruction with a more practical, investigative approach.

There is a good reason to conclude chapter 5.5 by saying that Øystein and his class have given a kind of existence proof for the potential influence of KappAbel: It may change – or at least influence - teachers' and students' views and practices. Øystein and his students had an invitation and a challenge from KappAbel, which they accepted. From that they have learnt a lot – e.g something about what mathematics also might be and how you learn mathematics.

Chapter 6

Mathematics teaching at Bjerkåsen Lower Secondary School: The case of Kristin and her students

In the autumn of 2005 we[1] visited a school in the southern part of Norway. The school, *Bjerkåsen Lower Secondary School*, was selected because one of its classes had made it to the national semi-finals of KappAbel in 2004/05. Also, Kristin, the teacher of the class in question, had responded to the second questionnaire, and accepted being contacted for interviews and observations.

In this section we shall analyse the practices of mathematics teaching and learning in Kristin's classrooms. We shall supplement our analyses of the classroom interactions with her response to TS 2 as well as with analyses of the interviews conducted with her and her students during our stay at Bjerkåsen. Our modest ambition is to shed some light on if and how KappAbel plays a role for how Kristin and her students engage with and conceive of mathematics, i.e. for the didactical contract in the metaphoric sense in which we use the term (cf. section 2.3).

It should be mentioned that we do not expect Kristin's situation at Bjerkåsen to be in any way indicative of the type of influence to be expected in most Norwegian classrooms. There are two reasons for this. First, the school has a strong tradition of participating in KappAbel. This is a tradition that involves all the mathematics teachers. Also, among most members of the mathematics department there is a strong sense of community and of mathematics teaching as a joint and collaborative enterprise. This does not mean that they normally share their teaching obligations by co-teaching. But it means that they take joint initiatives to develop mathematics education at the school, for instance in organising "days of mathematics" for particular grades or classes. This collaborative spirit may be seen as part of the background of the school's tradition of participating in KappAbel, while this very participation in turn appears to have further fuelled the sense of community. The fact that one of the classes, 10 Q, made it to the national semi-finals in last year's KappAbel has

[1] 'We' here refers to the research assistant, Kjersti Wæge, and Jeppe Skott.

further contributed to making mathematics education a common enterprise at Bjerkåsen.

Second, the school is located in an affluent neighbourhood, and many of the parents have strong educational backgrounds. Based on the general relationship between parents' educational level and their children's school performance, not least in mathematics (e.g. Kjærnsli et al. 2004, s. 205), one may hypothesise that children in this neighbourhood in general outperform most of their peers. Further, based on studies like e.g. Lubiensky (2002) one may speculate that this is not less so in the type of open and investigative setting envisaged by KappAbel.

The case of Kristin and her students, then, is to be thought of as a critical, rather than a typical case (Patton, 2001; p. 236 ff.). This means that we do not expect the situation at Bjerkåsen Lower Secondary School to be in any way exemplary for Norwegian schools with regard to the potential influence of KappAbel. Rather we expect that any influence, or lack thereof, of KappAbel at Bjerkåsen may be indicative of the limits of the possible influence of the competition. In other terms, the case of Kristin and her students should be considered as one of KappAbel's potential influence under conducive circumstances in terms of positive traditions and experiences with the competition and of the social status of the neighbourhood.

This wording does not do justice to the possible multi-dimensionality of KappAbel's influence. This means that our findings are not only to be considered special in terms of possible quantitative aspects of the influence of KappAbel, but also in terms of the more qualitative ones: there is no need to assume that influence of KappAbel is different only in degree under other circumstances; it may very well be different also in kind.

6.1 Data and methods

During our four-day stay at Bjerkåsen, we collected data on Kristin and her students in a number of ways. First we spent altogether 10 mathematics lessons with Kristin in 3 different grade 10 classes. All but one of these lessons were 'part-class lessons', i.e. lessons in which Kristin taught only part of a class, normally (but not always) approximately half of it. This is possible, because extra resources have been made available in order to ensure that there is more time for Kristin to support the student's learning. All the lessons were video recorded. On the same day we watched the recordings and used them to elaborate on the field notes made during the lessons. Later the recordings from two of the lessons were transcribed in full. These lessons, one whole-class and one part-class, are both with 10 Q. Subsequently the transcriptions were analysed without a pre-developed set of codes. We used constant comparisons in a way somewhat inspired by grounded theory (Glaser & Strauss, 1967), but without the objectivist assumptions sometimes ascribed to this methodological stance (Roman & Apple, 1990). In other terms, we were inspired by the technical methods of grounded theory without buying into some of the basic epistemological understandings sometimes associated with the methodology. The main intention was to view the classroom from the perspective of how the participants' contributions to the interactions contribute to the emergence of a dominant set of practices, primarily as they relate to mathematics teaching and learning.

Second, we spent one day with Kristin and her colleague in a grade 9 that was doing a 'day of mathematics' as preparation for their participation in this years' KappAbel competition. Although this was a special KappAbel-event, it was not planned for us. On the contrary, Kristin was initially sorry to tell us that she was not teaching her grade 10s as usual on that day, because she was to support her colleague teaching this years' grade 9. Considering the limited time available for us to spend with Kristin in her normal classes we also initially regretted that this was the case. However, it turned out to be quite an opportunity for us to get to know how the school had developed a tradition of preparing for participation in KappAbel.

Third, we conducted interviews with Kristin and her students. Immediately upon our arrival we had a 30-minute informal talk with Kristin, during which she introduced us to the ways she handled her teaching obligations in general. This was supplemented with numerous talks during the week, in which we informally discussed our impressions of Kristin's teaching. Towards the end of our stay we conducted a 30-minute formal interview with Kristin, which was audio-taped and later transcribed in full. Also, we conducted a group interview with six students from 10 Q, which was also transcribed in full, before it was analysed.

In this section we shall portray Kristin, some of her students and the practices that unfold in their classrooms. In line with our previous description (cf. section 2.2), we shall adopt a perspective of mathematics classrooms as fairly stable and robust structures and try to entangle the mutual expectations that have developed between Kristin and her students in terms of how to engage with mathematics during mathematics instruction with a special view to how these expectations relate to the priorities of KappAbel.

6.2 Kristin and the classroom practices

Kristin is a teacher in her late 40s, who has more than 20 years of teaching experience. Mathematics is her favourite subject, even though she does not have a strong academic background in it and does not identify herself as a mathematician. Instead she identifies strongly with being a teacher of mathematics and with being a teacher in general. Explaining why she likes teaching mathematics, Kristin says in the questionnaire: “The reason is that maths is a subject that many can master if they follow my presentation”. This wording appears to reflect the main emphases in what she considers relevant school mathematics and what she prioritises in terms of her own activities and those of her students in the mathematics classroom.

There are two immediately striking features of the interactions in Kristin’s classrooms. First, the classrooms are characterised by a very high level of activity, a drive that by now appears to have become the expectation of everybody involved. Kristin is extremely influential in this. She moves energetically and at a high speed back and forth in front of the board or around the classroom in all whole-class sessions, asking fast and short questions and expecting fast and short answers. And she gets fast answers. By doing so, she dominates the lessons in an insistent manner, both when asking questions for the students to respond to and when explaining or elaborating procedures or concepts. Kristin’s drive to a large extent contributes to the momentum of all the lessons observed, a momentum that is maintained not least because it has now become part of the mutual set of expectations between her and the students.

Second, there is in all the classes a very inviting atmosphere. Kristin also plays a strong role in this, but it has by now become a way of being together in the classroom that seems to involve everybody. Kristin keeps a constant smile, if not on her face, then at least in her eyes. In almost all situations she gives positive feedback in the form of praise of the students’ responses and solutions or of the way they otherwise conduct their ‘studenting’. The number of positive evaluations and exclamations is remarkable. Consequently, the dominant atmosphere is one that signals that the presence and participation of everybody is valued.

Analysing the interactions in Kristin's classroom, we shall initially follow what appears to be the general structure of the classes. They generally consist of a whole class introduction of variable length. This is followed by individual/pair work on textbook tasks, which is interrupted from time to time in order for the whole class to get back together. We shall exemplify the interactions in the two parts of this structure and make preliminary analyses of the activities and interactions in the process.

Setting the stage: whole class introductions

The lessons generally start off with a whole-class session. First a few minutes are spent to ensure that everybody settles down and feels at ease. Then key concepts and procedures are introduced or revised both as an important activity in their own right and in order to initiate subsequent student work. In the lessons observed, the introductory episodes are all in part retrospective in the sense that they refer back to previous work done by the class. This is the case not only when it is basically a revision of concepts or procedures dealt with before, but also when new content is introduced. An example of a revision occurs in a class working on construction of triangles. In her introduction, Kristin makes it explicit that they are to consider five such constructions, which "are the starting point for *all* the constructions we are to do from now on" (emphasis in original in the sense of a perceived stress in Kristin's pronunciation of the word or syllable in question; this is the way in which we shall use emphases throughout the chapter.). These triangle constructions are the ones in which (i) all three sides are known, (ii) one side and its adjacent angles are known, (iii) "we know AB, we know the angle A, and we know the distance from B, that is the distance BC", i.e. two sides and an angle not lying between them are known; (iv) "they come as something like this, AB is 8,0 cm, C is 4,0 cm *from* AB; and then it says that AC equals BC", i.e. an isosceles triangle in which the baseline and height are known; (v) one side, one of its adjacent angles and the height on the known side are known. Kristin also says that they will go over it quickly, as they have done it all before. Making her concise explanation on the fourth construction, she says:

> Kristin: In the fourth model […] hmm, AB is 8,0 cm C; C is 4,0 cm *from* AB. And then it says that AC is equal to BC. This is a frequent one. What is that I have to

remember, when I see that AC equals BC? What is it, then, that has to pop up in my head?

Student: An isosceles triangle.

Kristin: An *isosceles* triangle, super! Well done. And when it says ‘the distance from’, what is it that I use to find distance from a line?

Several students: [some mention ‘parallel’ others ‘perpendicular bisector’]

Kristin: Parallels. And because//somebody else said that, what//do I need to construct a parallel line in this case?

Several students: [inaudible]

Kristin: Why don’t I have to?

Several students: [inaudible, but perpendicular bisector is mentioned].

Kristin: It works with the perpendicular bisector, because at once I see that the two lines are the same length, and I can have the *distance*, then I am really lucky, then it works with the perpendicular bisector.

Student: [Asks if it is wrong to use parallel lines; exact wording inaudible].

Kristin: No, but you have much more work to do then, because then you *have* to construct the perpendicular bisector in the end.

Student: Yes.

Kristin: Then you get three tasks [constructing the parallel in two steps and making the perpendicular bisector afterwards], but it is not wrong. So if we take this briefly [makes a quick sketch on the board while explaining], then we have a long line, marks AB, constructs the perpendicular bisector and measure how many centimetres up here it is.

A somewhat similar situation occurs in a class working on scales. Kristin initially asks the students to recapitulate their previous activities on the topic:

Kristin: What did we do last Monday? It is a long time since we have been together.

Student: Scales.

Kristin: Scales. What do we remember about scales?

Student: [I] remember that we must multiply by the scales.

Kristin: Lovely! *Always* multiply by the scales. But what type of problem do I have, if I say that the scale is four to one [writes 4:1 on the board]?

Student: You’ll reduce it [inaudible]. No … [several students discuss, some claim the copy will be bigger].

Kristin: Can I//

Student: It'll be enlarged, because you take 4 divide by 1.

[Several students speaking at the same time; inaudible]

Kristin: Oscar, can I multiply by that, by the expression over there [pointing to the '4:1' on the board]?

Oscar: No, it is division [inaudible]

Kristin: What did I do? Torsten?

Student: Multiply by/

Torsten: Divide it.

Kristin: Yes, I divide it, what goes on top?

Torsten: Four.

Other student: Four fourths.

Kristin: Four divide by …?

Torsten[?]: Four divide by 1.

Kristin: Lovely! And the answer is?

Several students: Four.

Kristin: Do you remember now?

A situation in which the reflection on past activities is to take the students into new topics occurs in an episode in which Kristin introduces point symmetries by referring to work done a few weeks before on line symmetries. She draws a line and a triangle on the board and asks the students how to draw the reflected image of the triangle. Informally they discuss briefly "what it looks like on the other side". Kristin continues: "What we are going to practise now, is to reflect in a point, 'cause we haven't done that before." Next the whole-class setting is used for a revision of how to properly construct the line symmetry of the triangle already on the board. Following from there, Kristin asks the students to draw another triangle in their notebooks and make a point, P, outside the triangle. In what follows, they individually, but as part of the whole-class setting, follow Kristin's presentation of the process of making the image of the triangle as reflected in the point:

Kristin: Select one of the points, you'd better take B as I am at B on my drawing. Draw the line from B through the point P, oops, that is difficult on the board [she does it on the board, and the students follow the instructions in their notebooks]. […] From B through P. OK?

Student: [Inaudible, but to the effect of whether it can go through B]

Kristin: Yes you can//whether it passes through B, that doesn't matter. […] The point is that the ruler must have those two points for us to know where to draw the line. Then you take the pair of compasses, place it at P, and measure how far it is to B … [she does it on her own drawing] Then you turn around and mark off B'.

The introductions last anything from 7-8 minutes to almost half an hour in the 45-minute periods we observed. The length of the introduction depends on the character of the difficulties the students appear to have with the concepts or procedures in question. It does not, however, depend on whether it was primarily conceptual in a general sense or more task oriented; neither does it depend on if it is a revision or a presentation of new material. It is another and more significant characteristic of the introductions, that they often include a combination of conceptual and procedural emphases with the latter being the dominant one. This is apparent also in the three episodes above. In the first episode, for example, the class may be seen as discussing different types of triangles and the information sufficient to construct them. In this transcript, the initial conceptual emphasis on isosceles triangles shifts towards the operational aspect of a way of constructing them. In the interchange this in practice becomes *the* way of constructing them. In the second episode the apparently open and potentially conceptual question of "What do you remember about scales?" prompts a primarily procedural response about the multiplicative character of the concept, a response which in turn is reiterated by Kristin. In the same episode the discussion on whether to multiply or divide is one that is fuelled by a miscommunication about the scale of '4:1'. Kristin's intention at this stage is apparently to have the students perform the inherent division, i.e. to conceive of the scale as an operation to be performed (4:1 as 'four divided by 1'), while some of the students are considering the scale a fraction, a number to be operated on. In both cases, however, the utterances emphasise the operational aspects of the concept.

Responding to students' questions and comments

The students' role in the introductory phase is primarily one of responding briefly to the questions Kristin poses. These questions are often closely linked to procedural aspects of the topic and are even more often conceived as such by the students. This is generally the case also with the questions that the students work with in subsequent individual and whole class sessions. We shall exemplify this with three more episodes.

In the first episode cited above, a student suggests using parallel lines to make an isosceles triangle. Kristin accepts that this is an option, but in practice turns down the suggestion, because it is more cumbersome than measuring on the perpendicular bisector. However, in the subsequent work on the fifth triangular construction, i.e. the one in which one side, an adjacent angle and the height are known, Kristin decides not to go over the construction on the board, but to refer to the suggestion of using parallels as a way for the other students to make the construction themselves. She says:

Kristin: […] In model five we are told for instance that AB is 9,0 cm, we are told that this time B is, oops [makes a mistake on the board], is 60 degrees. And then the angle C is in eh, five cen//or angle C is five centimetres from AB. Make a draft drawing of that one, then I shall walk around and see if everybody gets it … You got a good hint from Erik for this one.

[…]

Kristin [now working with a student at her table]: And as you have the line, you can measure along that line. And then you make the 90º in those two points [referring to A and B]. Don't make new points, you see?

Student: Yes.

Kristin: Make//there, yes. [Referring to the points A and B where the student is to make the right angles]. [In a louder voice, for everybody to hear:] Make the 90º in the two points that you have on the line already. [Turning to another individual student] Yes, that is perfect. And then you make this a dotted line, don't you [The line connecting the end points of the 5 cm perpendiculars made at A and B]?

Kristin [working with another student]: Nice, very nice. .. Mmm [supportive]. Nice. But you, you are miss//you haven't got it yet. And the reason you haven't got it is that you have made a perpendicular at A and made a perpendicular at B. Measure 5 centimetres and dot the line across. Where that cuts the leg of the angle, that is where C is. […] [To the whole class] Shall we do this together [this is not a question]? It looked really good with all of you [briefly goes over the construction on the board. Continues] How can I construct a parallel without making two perpendiculars? How can I get the last point more quickly? Do you remember? We haven't done it for at long time. Yes?

[A suggestion from a student]

In this episode Kristin supports the students in constructing the triangle in question. She does so by referring to Eirik's suggestion for the previous construction of drawing a parallel line. However, she initially leaves it to the students to do the construction themselves. Some of them immediately manage to do so, while Kristin provides very clear instructions to the others on how to proceed.

Both in individual, small group and whole class sessions problems come up, which are not immediately related to the topic in question and therefore unexpected or at least unplanned for. This may be exemplified with an episode that occurred a few minutes after the second transcript in the previous sub-section, i.e. the one in which Kristin reviews the previous work on scales. Kristin refers to a rectangle she has drawn on the board and says:

Kristin: […] Now we say that this is on a scale of//now we are going to make slightly more difficult. We say on a scale of 2:6. [She writes 2:6 on the board]. We jump directly to something slightly more difficult… What can I do with this?

Student: [Short inaudible]

Kristin: Use your calculator.

Student: Zero point three, three, three.

Kristin: All of you, use your calculators.

Student: Zero point three, three, two.

Kristin: Zero point …?

Student: [inaudible] In fact, it is thirty-three, but if you are to take it down then it is thirty-…five …

Kristin: Oscar […]

[…]

Oscar: Zero point thirty-three.

Kristin: Yes, zero point thirty-three. Reduce it first, just for us to remember. Just reduce it as a fraction.

Student: [inaudible] divide by [inaudible].

Kristin: And then it is ….?

Student: A third.

Kristin: Lovely. And that is zero point thirty-three [continues writing on the board: $2:6=\frac{2}{6}=\frac{1}{3}=0{,}33$]. Then I can multiply by zero point thirty three.

In this episode the students' attention shifts from solving the initial task, which is related to scales, to one of rounding off decimals. Kristin follows up on the decimal issue immediately after they have solved the task:

Kristin: Lovely. This is great, what you are doing. Bodil was a little uncertain about something. Bodil was a little uncertain about something. 'Cause she says that it is zero point three, three, but really it is zero point three, three, three, three, three, three. And then she says that when you round off, zero point//then I took three, and then I have to round it off to that one. Then Bodil said that it was zero point …

Student: thirty two.

Kristin: yes.

Several students: [Inaudible]

Kristin: Yes, what is the rule?

Student [speaks loud to be heard above the others]: If it is below five, then you have to leave it with two decimals, but if it is five or more then it has to be bigger.

Kristin: yes, that is very good, you remembered the rule.

[…]

Kristin: […] you remembered the rule, 'cause you remembered that it was something//if it had been zero point three, three, five, then the five and had changed that to zero point thirty-four, when it is rounded off, because five, six [inaudible] how does it go, I have to include the five, six, seven, eight, nine they round up; but if it is thirty-three, then it doesn't round down, it keeps …

Student: Yes.

Kristin: Do you remember now?

In this situation Kristin manages to pick up on the student's difficulties with rounding off decimals. One of the other students comes up with a description of how it should be done, and Kristin praises him for 'remembering the rule', which she then repeats.

Episodes that are similar in that the students' focus on the initial task is changed by their difficulties with operations that are necessary for solving the task, but that are not an explicit part of it, include changing between units of measurement for area (e.g. m^2 to cm^2) and operating on fractions. In these other situations a rule-orientation develops, which is similar to the one in the above episode.

Most of the tasks that the students solve after the introductions are very closely related to the tasks and procedures that have been dealt with in it. The introductions, then, in a very immediate sense prepare the students for their subsequent individual or small-group work. For instance, the work on the construction of triangles referred to in the second episode in the previous subsection is followed be student activity on exercises like the following:

Example 1:
In triangle ABC, AB = 6,5 cm, $\angle$ A = 45º, and $\angle$ C = 90º.

a) make a draft drawing and construct the triangle
b) Reflect the triangle in the AC and call the reflected image of B for B'.
c) Calculate the area and the perimeter of the figure ABCB'.

The task of the student in cases like this is to recognise the information given as an example of one of the five constructions previously dealt with and use the appropriate standard procedure.

Sometimes, however, the textbook tasks involve a greater level of complexity than practising what is a standard procedure in the sense of one that has been presented in advance to be used in subsequent student work. For example the section on scales includes the following task:

Example 2:
A factory site has an area of 12.000 m^2. What is the area on a map drawn on a scale of 1:1.000?

The intention seems to be to challenge the students with a task that requires them to do more than just use the examples provided by the book. Introducing the task Kristin says:

Kristin: Wow… Now we have a problem, 'cause now it is suddenly quite different from what we've done before, isn't it? Yes. Well, let's do another task first. Now we shall do a task that is a little easier first, and then we shall see if you can manage somehow. First we do a task that is a little easier. Well, let's have a go at … 4 times 2. […] Now we take this rectangle here [draws a rectangle on the board and writes '4,0 cm' and '2,0 cm' on the sides]. And we say that this is four point zero centimetres and this is two point zero centimetres. And then we multiply to get the area. What is the area of this rectangle?

In what follows Kristin takes the students through the task of finding the area both of the rectangle she has drawn on the board and of a copy of it drawn on a scale of 2:1. The area of the copy is found by first finding the length of the sides. They end up with the answers of "8 cm^2" and "32 cm^2" written in the two rectangles. She continues:

Kristin: […] what happened now? We have doubled it haven't we? We have *doubled* the side lengths. What happened to the relationship here? Can you see how many times bigger the area has become in comparison with that one?

Karina: [inaudible].

Kristin: [inaudible] Can you see that it has become four times bigger [to the other students; probably confirming Karina's answer]? Great Karina. Can the rest of you/how did she find it?

Student: [inaudible] divide 32 by 8 and that is 4.

Kristin: That is 4, so it is four times bigger. But on what *scale* was it//On how much bigger a scale did we draw it?

Student: Two to three?

Kristin: Two times bigger, didn't we? Do you see that the area was//How many times as big?

Student: Four.

Kristin: Four times as big.

Student: So it was double the scale!

Kristin: It was *double* the scale. So if I am concerned with the area, I can in fact take//go straight to the scale and take//sorry, two over one and multiply by two over one. Four over one or four [writes $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 4$]. Then I can multiply the area directly by the two scales multiplied by one another and get the area of the next one. This here is a little dangerous, for all of a sudden it may go wrong, and the easiest way to do it is to go via the lengths. Kjersti?

Student: Just multiply the scale with the scale, then?

Kristin: Yes. And then we get what we can multiply the area with, or divide the area by.

In this episode, Kristin takes the students through the observation that when a copy of a rectangle is drawn on a scale of 2:1, the area of the copy is four times as big as the original. She apparently wants to build on that observation in the next task. She continues:

Kristin: […] Now you try this task [example 2 - the one with the factory site] by using//by not using the areas first, use the lines. What is it that we have to find? Write down what I did [inaudible] write down ’12.000 m^2’ … and ‘Scale 1:1.000’. What does the scale of 1:1.000 mean?

[a few minutes interaction on the meaning of 1:1.000].

Kristin: Right! You were very good. You were really clever. Very clever [joint laughter]. Lovely. So do you see that 1 centimetre is 10 metres? Good. Now I am not going to say anything for a while. Now you are to see if you can come up with something smart. Do it either way//I shall help you a little. Either you can come up with something that will help you come up with the answer and use the lengths. Otherwise you use what we did over there [pointing to her example on the board]. But I realise that that is a bit difficult (laughter).

Student: [Inaudible]

Kristin: What/Should we do what we did last, then, ‘cause that may be easiest to remember? […] What is one to one thousand as a fraction?

Student 1: [inaudible] one thousandth.

Kristin: What did I do with the scale, then, last time?

Student 1: [inaudible]

Kristin: Yes I did, didn’t I? I multiplied by each other, didn’t I? Try, then, to see what happens.

Student 2: [inaudible] help us too much.

Kristin: Yes, I do help you too much, yes [laughter].

Student 1: What did you say?

Kristin: Ok, then I am not going to say anything. I *said* that I shouldn’t say anything, and then I couldn’t help it.

Student 1: But [inaudible] did you have to multiply?

Kristin: Over there [pointing to the blackboard] we managed to find that if we had a scale of 2, then we enlarged, twice as big, the lengths. But the area became four times as big. And the way we found that it was four times as big was to multiply scale by scale … OK? And then we could multiply by four. See if you can take scale by scale now and see what you can do with it.

Many students continue to have problems with the task, and they become involved in a type of individual-in-whole-class activity in which they are struggling to find solutions to the task individually or in small groups, but at the same time asking questions aloud for everybody to hear. Kristin responds by addressing the questions of the individual students, but also speaks so loud that her comments may function as assistance to everybody in the class. This apparently prompts a student to say that she helps them too much, a comment that refers to her explicit intention of not helping them too much in this. In spite of the students' comment and of Kristin's acceptance of the point, the same type of communication continues over the next few minutes. Kristin tries to help the students complete the task by addressing the problems immediately related to the procedure she has suggested, i.e. one of multiplying the scale by itself and multiply the result by the area of the site. She sums up the procedure in the end:

Kristin: When we did/when we doubled here, then we get that the area is four times as big. But the scale was 1:2. And then I showed you that there was a way, so that we could go directly from the area to the next area, the scale multiplied by the scale. Take 8 and multiply by 4. Then you have the area. In the task you got now, the area in real life was 12.000 m^2, the scale was 1: 1000. Then I taught you, but you have only heard it once, so this is difficult, that you could take the scale and multiply by itself once more. Then I can go directly to the area. Then I take one thousandth multiply by one thousandth. Then I get one millionth, don't I? Difficult numbers! I did that. And then I said/then *you* said [addressing a particular student] that we can always multiply by the scale. […]

Following from that the question arises of how to rewrite 12.000 m^2 in cm^2. This question is clearly related to the one of the scales, but still beyond the wording of the initial task.

Kristin: […] How many places do I have to move the decimal point, how many zeros do I have to add in order to get to square centimetres?

Student: [Inaudible]

Kristin: How many for each unit of measurement?

Student: I don't remember.

Kristin: You don't remember? Then we just do like this, then we remember [draws this table on the board].

	m^2		dm^2		cm^2	
120	0	0	0	0	0	0

Look here now. Square metres, square decimetres, square centimetres [points to the table]. Now, we had 12.000 m^2, do you agree [fills in 120 and the two zeros under m^2 in the table]. To get to square decimetres, how many zeros do I have to add?

Student: [Inaudible]

Kristin: Lovely! Christina, in order to get to square centimetres, I have to add …?

Student: [Inaudible]

Kristin: Brilliant! Now I have a big number. Now I have to get it right, 12000 m^2, that is 12// is 120.000.000 cm^2. What is the rule for multiplying a number with a fraction?

Student: Multiply the numerator and keep the denominator.

Kristin: We are good at that. Super, well done! And then we get?

[…]

Kristin: And we have 120 cm^2. We could have done it without all the things we did now. We could have found two numbers which we multiplied to get 12.000 m^2, and that may be the easiest. […]

[They continue going over the task using the lengths to find the answer]

The lengthy transcriptions above are selected in order to show aspects of mathematics instruction that we find indicative of the teaching-learning practices in Kristin's classrooms. Apart from the previous points of the short questions and answers and the constant praise of the students' performance, there are two further interrelated issues that appear to play a strong role in the teaching learning processes. These are concerned with what we shall term the rule-bound aspects of the instruction and the sequencing of the instructional tasks.

Relying on rules

The transcripts above exemplify that the students are often invited to come up with suggestions for procedures or definitions. Also, they are sometimes asked to reflect on each others' suggestions and take them as starting points for their continued joint or individual activity (e.g. lines 133-137). However, both the students' contributions and Kristin's responses and explanations are primarily related to the procedural aspects of the task in question. This leads to what we conceive as a dominant rule-bound character of the instruction in Kristin's classroom.

Sometimes the focus on rules is apparent, as when Kristin explicitly asks for a rule for performing a certain operation. For instance she asks the class for a rule for how to round off decimals (lines 110-121) and how to multiply a whole number with fraction (lines 209-211). However, a similar emphasis on rules for operating on mathematical objects also characterise both the students' and Kristin's contributions to other episodes. These include for instance, when:

- the students' answer to the question of what they remember about scales is that they must multiply by the scales; Kristin responds "Lovely! *Always* multiply the scales" (lines 26-29);
- a student sums up Kristin's explanation about the relationship between scales and area by asking "Just multiply the scale with the scale, then?"; Kristin responds: "Yes. And then we get what we can multiply the area with" (lines 152-153).
- a student does not remember how to change between different units of measurement for area, and Kristin makes a table that shows how it may be done (lines 193-205).[2]

A focus on rules may function as a summary of a less structured investigation, as a reification of an investigative process. However, in most cases the explicit or implicit emphasis on rules appears to carry the assumption that knowing mathematics is most significantly a matter of doing it, and that doing it is primarily a question applying pre-developed procedural prescriptions. Consequently, the students' mathematical performance is closely related to their mastery of the rules.

The rule orientation of most of the interactions in Kristin's classroom may seem to be at odds with the expectation that the students are to contribute by presenting their own observations and suggestions, sometimes as a response to open questions or comments. A further analysis of a few of the episodes may shed further light on this apparent ambiguity.

In lines 130-153 Kristin initially invites the students to think of what happens to the area of a figure, if its side lengths are doubled. When Karina answers, Kristin asks the rest of the class to consider how she found the result of "four times as big". The student who responds to this question focuses on the immediate division of the areas of the two figures ("divide 32 by 8 and that is 4"). Kristin attempts to shift the students' attention to a comparison of the

[2] Other similar episodes include the ones in lines 54-58, 64-70, and 146-149, 197-203.

relationship between the areas and the relationship between the lengths, i.e. the scale. A student observes that the former relationship is double the latter (line 145). Kristin builds on that observation in her subsequent comment, but apparently realises that the student may be suggesting that the relationship between the areas is always double that of the relationship between the lengths. She continues by implicitly correcting the possible mistake and explains that you need to multiply the scale by itself. However, she immediately discourages the students from doing so, as it may go wrong (lines 149-151). By doing so she implicitly points out that the students are not required to work with the areas by squaring the scale, but may use the side lengths one by one. In a later comment to the episode, Kristin points out that it was important for her to make that point, but that most of the students decided work directly with the areas anyway.

Adopting a process perspective on mathematics, this episode may be interpreted as an example of a student making a bold conjecture based on a single observation, namely that the relationship between the areas is double the scale. The conjecture is taken up by the teacher. In the process she tries to avoid what may be seen as a possible misunderstanding on the part of the student who made the conjecture or on the part of other students. The result is a wording of a rule for how to work on areas based on the knowledge of scales, a rule that is a generalisation the students' conjecture, but one that does not explicitly address the possible misunderstanding. The teacher closes the episode by introducing a 'meta-rule' of not using the rule of squaring the scale.

Sequencing instructional tasks and questions

As mentioned previously, the introductory sessions serve - among other things – as a very immediate preparation for the students' independent work on tasks similar to those dealt with in the whole class setting. Most of the students' independent work, then, becomes of a routine type in the sense that it is a field for the application of concepts and procedures that have been dealt with and practised in advance.

This routine character of the teaching-learning processes in Kristin's classrooms applies also to situations in which the students work with tasks that may challenge them differently if used with less explicit procedural instruction. To illustrate the point we shall return to the task introduced in lines 122-129, i.e. example 2 on finding the size of a drawing on a scale of 1:1.000 of a factory site with an area of 12.000 m^2.

Introducing the task, Kristin immediately claims that this is a difficult one, and that they need first to do another simpler one to overcome some of the inherent difficulties. This leads to the interaction in lines 130-153 that was discussed as an example of the rule-bound character of the teaching-learning processes in Kristin's classroom. In what follows, Kristin attempts to devolve responsibility for solving the initial problem to the students (line 161-165). She does so by suggesting that they either come up with 'something smart' or work directly with the areas, i.e. use the method she has just shown. However, she immediately resumes at least partial responsibility by suggesting that in spite of what she said previously, they should use the easier example as a template. She subsequently takes the students through the process of solving the problem by asking fairly closed questions like "What is one thousandth as a fraction?" (line 168) and "What did I do with the scale, then, last time?" (line 170). The episode ends by Kristin recapitulating the procedure to use (line 187-192).

Although Kristin says that this task presents a different set of challenges from the other ones the students have encountered, she is not explicit what the differences are. However, they seem to include that the students have so far not been asked to consider how the size of a figure depends on the scale on which it is drawn. Also, the information provided in the previous tasks includes the shape of the figure in question. This information is left out in the current task.

The episode exemplifies a double tendency towards structuring the students' involvement with mathematics by subdividing and ordering the problems they are to work with. First, Kristin inserts and sequences tasks so that simpler ones are dealt with first in order to avoid insurmountable difficulties for the students. This means that the original problem of finding the size of the drawing of the site is transformed into an exercise-type of task, i.e. an application of a procedure, which the students have already seen in use in the simpler example. Second, the questions that are posed and answered structure the students' engagement with mathematics when dealing with each of the tasks. The closed questions mentioned above and the way they are sequenced insert a stepwise pattern into the solution procedure, which further contributes to turning the original problem into a routine-like operation.

There are at least two ways of viewing this episode as an instance of an attempt to facilitate students' mathematical learning. On the one hand it may be seen as an example of the teacher seeking to keep the focus on the key product component of the task, i.e. the relationship between the scale and the relative size of the areas, while ensuring that the students overcome some of the cognitive and affective difficulties she expects them to face when solving it. In this interpretation, she introduces the procedure with the triple intentions of maintaining the mathematical emphasis of the episode, avoiding the students' getting stuck in technical difficulties of for instance working with fractions like a millionth, and shunning the threats to the students' self-confidence that may arise when they work with tasks with which they have no prior experience.

On the other hand, the episode may be read as an example of the teacher replacing a problem with a routine task and dismantling a significant part of the learning potential in the process. From this perspective, the main intention of the task is neither to have the students come up with a solution nor to have them practise a given solution procedure. Rather, the task is an invitation to develop one or more ways of addressing the problem, ways that may be discussed, refined and further developed into a more general method applicable also to other similar situations. This is a rather more process oriented understanding of the task than the one inherent in the above interpretation. The process perspective requires the students to face exactly the types of challenges that the former approach tries to avoid in order to ensure the students' acquisition of pre-developed concepts and procedures as well as to sustain their mathematical self-confidence.

Presenting these two interpretations of the episode, we do not mean to imply that product and process orientations are incompatible in principle. Neither do we intend to suggest that – when they are not easily reconcilable - one is in all cases preferable to the other. Our intention is merely to point out how the sequencing of tasks and of questions within tasks seems to amplify the product orientation of the students' mathematical activity at the expense of the process aspects. The inserted tasks and questions structure the initial problem by subdividing it and ordering the individual parts so that students are taken step by step through the process of solving it.

6. 3 Kristin's view of school mathematics and the role of KappAbel in it

In TS 2 and in the formal and informal interviews conducted at Bjerkåsen, Kristin elaborated on her views of school mathematics. They may be characterised as comprehensive and multifaceted. In TS 2 she gives equal priority to the four items of question 9, i.e. the ones of everyday applicability, investigative aspects, logical reasoning and basic skills. Also, she agrees or agrees completely with all seven items of question 11 on the characteristics of a good mathematics teacher, and with all six items of question 13 on the characteristics of a good student of mathematics. According to Kristin, then, a good mathematics teacher is characterised for instance by her ability both to give short and precise answers and explanations to the students' questions (item 11.1) and to facilitate student involvement in mathematical investigations (item 11.4). A good student is similarly described as one who can – amongst others – both solve textbook tasks fast and correctly and raise new mathematical questions on the basis of the answers they have found.

However, this apparently all-encompassing view of school mathematics is somewhat challenged by Kristin's response to question 10 on the characteristics of good mathematics teaching. Asked to prioritise (1) students' investigations within mathematics, (2) their acquisition of routines related to the basic skills, and (3) their involvement in cross-curricular projects she points up the second option, by giving it 20 of the 30 points available.

The product orientation in this emphasis on practising the basic skills is reflected also in Kristin's key words on when she is satisfied with her own teaching. In full her comments on this situation read:

"Quiet; focussed; explain clearly what is to be learnt in the lesson; easily accessible presentation; practise, new information, practise again; the ones who can continue do so, the rest do the calculations with me". (TS2, item 12.1).

Characterising a good mathematics teacher in general, Kristin says:

"It is important to systematise the contents for the students and all the time show the connections between the topics. I think that the process of automation, knowing the rules by heart, is an important factor to gain understanding of the subject and continue building on one's knowledge and skills. Then they experience how systematic and logical mathematics is, and then it becomes fun" (TS2, question 11).

And describing when she is not pleased with her own teaching, Kristin says:

"When the structure and the goals of the lesson are not reached" (TS2, item 12.2).

The lack of emphasis on project work is elaborated later in the questionnaire. Kristin does not think that it is a good idea to do projects in mathematics apart from in KappAbel. The reasons are that project work does not allow the students to become absorbed in mathematics in its own right; that it does not allow sufficient emphasis on practising the skills; that there are other and better ways of relating mathematics to real world applications; and that it would leave too little time for activities more immediately related to tests and exams. However, Kristin strongly disagrees that doing project work would "provide the students with the wrong image of what mathematics really is".

Kristin considers it important that KappAbel engenders student collaboration: "That is the whole point of this type of test". But she also considers it an important aspect of mathematics education in general. Students should, she says, be allowed to help one another, and they may challenge each others' understandings, when they work together. But Kristin prefers that the students work in pairs. In that case they may

"both do the calculations and help each other on the way. They like to have things explained to them by other students. In larger groups some will become passive." (TS2, item 20.1)

Individual work is needed to some extent:

"Everybody needs different amounts of time in order to understand and routinise mathematics; the aim is that the students can perform the tasks without any assistance. Working alone is super, as long as you get assistance whenever you get stuck, either from a fellow student or from the teacher". (TS2, question 20.2)

Kristin modifies her point about the passivity of students working in larger groups by explaining that in KappAbel and other problem solving tasks she uses groups of 4 or 5. In projects she may use groups of 5-7 students, but as mentioned above she does not use project work except in KappAbel.

Kristin's comments on boys' and girls' abilities in and engagement with mathematics indicates that she does not consider gender a major issue in the subject. She considers their interest, effort, ability and self confidence in mathematics essentially equal. But she does consider it a nice feature of KappAbel that the groups are to made up of both boys and girls. This, however, appears to be due to a general concern that boys and girls learn to cooperate and behave towards one another, rather than an issue that is particularly related to mathematics.

In the interviews conducted at Bjerkåsen 10 months after Kristin filled in the questionnaire, she elaborates on her understanding of quality mathematics teaching. Asked to share her thoughts about the lessons that we observed, she says:

> Kristin: What do I like when I am//or what do I think? When I get the children turned on, when I get them really excited, preferably all of them. Then I find it fun. Then I am enjoying myself, and I can see that the students are enjoying themselves. In a way that is my main aim for *all* the lessons, that I shall make *everybody* wake up and turn them on.

On several occasions throughout the interview Kristin returns to the idea of having the students 'turned on'. Exemplifying what it means she – without being prompted - refers to the lesson dealing with example 2 in the transcripts above (lines 154-184), i.e. the one in which they use two different ways to find the area of a drawing of a factory site drawn on a scale of 1:1.000. The students' energy and enthusiasm when solving the task impressed her:

> Kristin: What impresses me is when I have given them a task that has two solutions […] then everybody makes sure that they have written down both ways on their paper//there was nobody who didn't try in that lesson.

Kristin implicitly describes the meaning of having "the students turned on" by referring to what it means in terms of the students' activity and what it takes for her to support it. She consistently refers to the students' need to practise basic and slightly more advanced mathematical skills. This is necessary, as there are "so incredibly many details that must get in// that have to get into [their] heads".

Asked if the students should ever be encouraged to find their own ways of doing things, Kristin in the first interview says that “the road will be too long to walk for those students who are not strong [in mathematics]”.

Kristin considers it important for her to develop a strong personal relationship with the students. She “thrive[s] on being with my angels [and] adopt[s] all the children in a very personal sense”. In terms of the more content oriented aspects of her job, she says that her obligation is to structure and sort the tasks, because “I *believe* in that, [for the students] to have the essence served at an instant”. According to Kristin, this is particularly important with the textbook scheme she uses. The scheme is ok, but one of its disadvantages is that it is not structured well enough, and consequently it is difficult for the students to use. In part this is due to the attempt of the authors to encourage differentiated teaching. The textbook uses different colours to indicate the level of difficulty of the different tasks, blue (the easiest), yellow and red. On the one hand there are too many exercises, as nobody will ever manage to do them all; on the other there are too few, as only doing one colour is not sufficient, neither in quantitative nor in qualitative terms. This means both that Kristin has the students do more than one colour and that she supplements the exercises in the book with other ones:

> Kristin: […] sometimes there are too few, but now I know that, so I copy sheets […] in order [for the students] to routinise it […] It is just not good enough, only to do the yellow exercises to know/be able to do[3] it.

Explaining why it is necessary to supplement the tasks in one section of the textbook with more ambitious ones, Kristin uses the example of the coverage of the Pythagorean Theorem. It is necessary for the students to know more standard results than the ones dealt in the easier sections:

> Kristin: Pythagoras in the blue [section] is just mainly to find the hypotenuse, and possibly something with just one of the other sides, but *only* whole numbers. Yellow starts off with whole numbers and then make two sides unknown, and use seven point five and numbers that are more difficult. In this book 30-60-90 didn’t come until red. It was only 45-45-90 that was in yellow. […] And that is a little//the last one should be for absolutely everybody, whereas those who only do yellow, that should be the normal, they lose *a lot* if they don’t do red as well.

[3] Kristin uses the Norwegian word ’kunne’. This may be translated either as ‘being able to do’ or as ‘know’. It is not clear which of these meanings is intended here.

According to Kristin, there are times and classes in which she has to be fairly explicit when directing the students' activity. This is the case for instance when there are big differences in the students' ability levels, and when she has new class that she does not know well:

> Kristin: [Then I need] to be more old-fashioned in my teaching, […which means] to have to go back to the blackboard, and then I *have* to repeat things. Then in the next lesson I have to repeat it again, and do it once more. 'cause they understand, but they forget so quickly.

It is important to Kristin, then, that the students develop proficiency in the basic skills and memorise key concepts and procedures. This is reflected also in her frequent use of home assignments and tests and in her attitude to the school leaving exam. The tests may be on a particular topic, in which case the students are told the topic in advance, or it may be on a broader set of issues. The home assignments are seen primarily as a way of refreshing and routinising topics dealt with previously. The students are given whatever help they ask for when doing the assignments, and they also have access to possible solutions to the tasks. As a consequence, they almost all get 4s and 5s (the highest grades). Kristin explains that the ones who get 4s have generally understood, but as opposed to those who get 5s, they may not have practised it all sufficiently. Kristin considers it important both with the tests and with the assignments that the students have their work returned within a day or two, so that can still recall their own thinking when they solved the tasks.

In general Kristin is not pleased with the school leaving exam. She regrets that there are generally no questions with a focus on the basic skills. It is dominated by problem solving, and it requires the students to read a lot. There is, she says, a dual problem in this. First it leaves students who perform relatively poorly in mathematics with no opportunity to show whatever they can do; second, even those who are good in mathematics perform poorly in the exam, if they are not also very good readers.

Asked specifically about the role of the KappAbel competition, she claims that it is very influential. This is so, not least as it provides support in some of the fields in which she had little experience beforehand. This is so especially when teaching probability theory and when working with problem solving. Kristin says that she uses problem solving a lot, especially in grade 9 as a preparation for the participation in KappAbel:

Kristin: … it is a process to have children not just copy accurately and then have the answer. But to dare to think, to dare to begin somewhere and dare to write and scribble and then tidy it up again and do it again and tidy up once more. KappAbel is fabulous at that, help to them cross that threshold, I think. It has been a great//and when you look at the exam, it is just pages up and down of reading. We talked about that. And that//then it is good to have done KappAbel to get used to reading, because a good deal of what we do to cover the syllabus is to routinise what is *inside*. […]

I: […] so it is not just because it is fun, it is also because you actually think that you prepare the students better for the exam by doing KappAbel?

Kristin: Yes absolutely. Because they get used to *reading* […] begin at the top read it through once, little by little, begin at the top and bother to read it through again. Of course it is a nightmare for those who are not happy with all the reading. […] But then the exam is a nightmare for the ones who don't like reading.

In the interview Kristin repeats a comment from the questionnaire, that they would never have done the project, if they had not made it to the national semi-finals. There simply is not time for that:

Kristin: I have no idea when I should have placed it [in terms of time]. *All* those hours that we spent after school. And they loved it. They gave up their [sports] training and all, they were here until late in the afternoon. You know that great big man […] They worked an they worked and they worked, and the booklet that went with it, and they worked and they worked and they worked, and they went on with the mathematics, and it was a *huge* job, fun, different, but we have//we cannot//it is too time consuming, that big a project, spending *so* much time on it. Three weeks in I don't know how many hours. […] But I had never done it, if we hadn't made it to [the semi-final].

That does not mean that KappAbel is not important, if you don't progress in the competition. But it means that to Kristin by far the most significant part of the competition is the problem solving tasks that you can use to supplement more ordinary mathematics teaching.

Kristin finds the mathematics department at the school wonderful. In her words, most of the mathematics teachers "think very much alike", and it is important also for the influence of KappAbel:

Kristin: I think that it is very important [inaudible] attitude that in grade 9 we do KappAbel. It is like//yes, it is sort of the attitude of the school that you are not left alone with it. I think that is *tremendously* important. 'cause it creates a sort of community among the students that they have been in this all together. I think it is really great that we do that here. Yes.

6.4 The students' views of school mathematics and of the role of KappAbel in it

During our stay at Bjerkåsen we conducted a group interview with six students from 10 Q, four girls and two boys. In the interview the students were asked to reflect on their experiences with school mathematics. More specifically they were to describe first what they consider an ordinary mathematics lesson and later what they think is particularly good one. Also the students were to outline their experiences with KappAbel, both with regard to the introductory rounds and to the project work in advance of the class' participation in the semi-finals. This included if and how the KappAbel activities differ from those that dominate mathematics instruction more generally.

The interview was audio recorded and later transcribed in full. Kristin had selected the students for the interview so that the group as a whole was to include students of different ability levels and so that none of the four students, who represented the class in the semi-final, was in the group. The latter issue was considered important so as to ensure that the students who had been part of the collaborative efforts of the class, but who had not had the obvious success of participating in the semi-final had the opportunity to voice their experiences with KappAbel.

It is a striking feature of the interview that there is almost complete consensus between the students on all the issues discussed. A few instances of apparent disagreement appear to arise mainly as elaborations of the points others have made, rather than as real differences of opinion. This is so in spite of the fact that all six of them are eagerly involved in the discussions.

Describing an ordinary lesson the students explain:

Martin: First she *says*, like what we are going to do in the lesson and she presents like//like says what it is that we don't know from before and explains it. And then we have to do some exercises on it.

Monica: She says like: *now we are going to learn something new*. You know, very concrete. Then she shows very carefully on the board. Then she says *Do an exercise*, one that has to do with that. Then everybody does *that* [exercise], and the ones who didn't get it, get help from Kristin, who walks around.

Ida: She is *very* good at explaining, and then it is really quick. She doesn't spend half an hour explaining something, it's like, shows on the board, and then nearly everybody understands [inaudible]

[…]

Nis: Then she goes over difficult exercises from the homework. And even if we do that, we have time enough for whatever we have to do in the lesson, because she does it really quickly.

Describing a particularly good mathematics lesson the students use phrases like:

Monica: That the teacher goes over the difficult parts. Or says: *now we are to learn this*, and goes over it […]

Kjersti: A good maths lesson is when she explains something that is difficult, and we understand quickly and then do the exercises, so that we get it routinised in the *first* lesson. I suppose that is the best maths lesson [inaudible] learn something new, and learn so quickly that after the first you know it by heart.

It is a striking feature that the students use almost entirely the same terminology to describe an ordinary mathematics lesson and a very good one. They are unanimous in their support for what they see as explicit expectations, concise explanations, exercises that are to the point, and a sense of immediate outcome of each lesson or instructional sequence. There is, then, common agreement that the tasks should have an immediate bearing on whatever contents Kristin has presented in her introduction. Also, everybody agrees that this is the case both with the textbook tasks and with the supplementary material that Kristin uses to supplement the book.

In spite of their emphasis on practising routine tasks, the students also agree that doing KappAbel is fun, although it is very different. The students all agree that being part of KappAbel and making it to the semi-final last year was a tremendous experience. It is apparent that the students especially identify KappAbel with the project work on *Mathematics and the human body* and to a much lesser extent consider the two preliminary rounds an essential part of it. And in the students' conception, the project work is really different from what they normally do.

Interviewer: When you worked with the project was that very different from the ways you are used to working?

Several students: Yes, yes. Very different.

Kjersti: We sat in groups//we worked in groups so that we could//

Monica: And then we looked for information on the internet and in books and other places. We didn't do like exercises like we normally do.

Martin: It was more like *thinking*. Like *how* do I find out? When we work in the lessons it's more like 'it is like this' and 'it is like this'. But [in the project] we had to *think*//figure out how to do the task.

Ida: And it wasn't just asking the teacher for an answer then either. We had to do it *ourselves*. We didn't get any help, so we had to do it on our own. [inaudible]. But it was fun. We could pick up whatever mathematics we wanted. It wasn't like: *Now we have to work with this*. It was just: *OK, what sort of mathematics goes into* [inaudible].

[…]

[one of the girls]: [the mathematics was] fun, a little difficult. Not too easy, but not just too difficult either.

[…]

Ida : And the mathematics could just get more and more difficult, you make it [inaudible] as difficult as you wanted, really, and work at your own level, and still try to find the most possible//the complicated maths tasks in order to make as much of an impression as possible

[…]

Martin: when we worked on the project, then *we* had to find the questions *ourselves* or the tasks ourselves, while in the introductory rounds we got//we *had* the questions, and then we had to find a way of solving it. While in the project we had to find question *and* solution, and for example *several* solutions, and more//in a way more *ways* to ask the question.

Asked explicitly about the preliminary rounds of KappAbel, the students say that they had prepared a very little bit for it in terms of solving some problems prior to the first round. One of the very few instances of disagreement between them concerns how well prepared they were for their participation in the first rounds:

Nis: It was like on that day, now we sit down in groups and then 'Now it is KappAbel'; yes OK!

Monica: OK, and then we did that (joint laughter)

Nis: I don't think we had much preparation.

[…]

Layla: I think we all//we were prepared for [inaudible] anyway, so we knew that we were to do it, but not exactly when. But we were prepared that we were to do it.

Initially, the students don't remember having done anything that is related to KappAbel between the two first rounds of the competition. On second thoughts, however, they agree to have done some problem solving on top of their ordinary lessons

Kjersti: […] we got like, it was sort of apart from KappAbel, sort of problem solving tasks that we had to do

Nis: But that was when we'd *finished* [what we were to do in] the maths book […

Addressing the question of if and how the way they worked in KappAbel resembled the activities in ordinary mathematics instruction, they initially agree that these are totally different enterprises. However, they reconsider and claim there were similarities anyway, at least in terms of the obligation to cooperate:

Martin: Well, you know, it was, we worked together, like. It was like *everybody* sat//the way I remember it, it is very much like//we had these groups, and everybody sat round a sheet of paper that had been passed around, and then everybody sat like that: "now I want to do it my way", and then they did it their own way, and then they watched a little "[inaudible] you can do this" and then you drew it together and found what you thought was the most correct way. And in a way that is what we do, in the part-class lessons at least, it is very much like cooperation, we watch one another, how did *they* do it. If you didn't understand it completely. […] And then you learn that that is the way to do it.

Following from Martin's explanations the other students join in and agree that this sort of cooperation dominates their everyday lessons as well, and are not limited to when they do KappAbel.

Considering the relationship between our interpretation of the classroom mathematical practices in Kristin's classroom (cf. the subsection on *Kristin and the classroom practices*) and the students' descriptions of ordinary and good mathematics teaching, it is to be expected that they are generally pleased with school mathematics. This is confirmed by the group interview. They consistently make remarks like "really good fun in fact, those lessons", "almost all the [maths] lessons are good”, “it’s like//the whole class is very much involved, it’s like everybody is happy with maths, I think”, and “I used to hate maths, now I really think it’s great”. These statements are indicative of the students’ comments throughout the interview, and it is obvious that they are extremely fond of Kristin and very pleased with her teaching. The point in the above interpretations of the group interview is that there appear to be at least two interconnected reasons for that overwhelmingly positive attitude towards school mathematics. First, the students are explicit that the personal relationship between themselves and Kristin in combination with her energy and catching enthusiasm is very motivating. This includes that the students are explicit that they are never told off for having performed poorly, and they are always praised for what they do. Second, both Kristin and the students value what has become their mutual expectations and modus vivendi in mathematics classrooms. This includes the role of explicit expectations and of the stepwise structure and sequencing of mathematics instruction. It may be surprising that in spite of those foci, the students still value what they consider the very different activities related to KappAbel.

6.5 Discussion – *change in views and practices*

Our intention in the previous sections has been to provide accounts of the practices in Kristin’s classrooms. We have described these practices as seen from our own perspective (the subsection on *Kristin and the classroom practices*), as well as attempted to view them from those of Kristin and her students. In the process we have attempted to incorporate aspects of how the practices and priorities with regard to school mathematics relate to those envisaged in the KappAbel competition. However, we have not analysed the practices of the classrooms and interpreted the interviews exclusively from the perspective of KappAbel’s priorities (cf. section 1.3). Rather we have tried to view the practices as social structures that have developed in the interactions between Kristin and her students, and interpreted the interviews as windows on the meaning that they attach to those interactions. The role of KappAbel, then, is viewed as the possible changes that may come about in the social fabric of classroom interaction by inserting KappAbel participation in what may be seen as the cracks and openings of the classroom practices (cf. Skott, 2005).

The most significant activities that mutually structure each other to create and sustain the practices that emerge in Kristin's classrooms are the mutually related ones of establishing good mutual relationships, providing and gaining speedy and easy access to distinct and identifiable learning outcomes, and preparing for the exam. These both lead to and are the result of Kristin's and her students' more specific ways of contributing to the classroom interactions. In particular their contributions are related to the ways in which:

- clear and brief instructions and explanations are given and received as part of how speed and momentum are built and kept;
- tasks are sequenced and are expected to be sequenced in order to keep the focus on identifiable mathematical outcomes, to keep the mathematical challenges manageable and to ensure an element of success for all;
- applaud and praise are given and acknowledged not merely as a recognition of progress in mathematical terms, but more significantly as a way developing and sustaining the atmosphere of the classroom;
- collaboration is organised as a way both to ensure a sense of communal enterprise and to sustain the drive established in whole class sessions.

These points may be read as if to relate exclusively to Kristin's contributions to the interactions. This is not the perspective adopted here. For instance, the clear and brief explanations and the way tasks are sequenced go hand in hand to create and sustain a rule orientation that has become part of the mutual expectations of how mathematics education is conducted; by now Kristin and the students alike contribute to maintaining this expectation. It has become part of the didactic contract in the metaphoric sense in which we use the term (cf. section 2.3). Similarly, applaud and praise does not become important merely because Kristin uses them as recognition of mathematical performance, but more significantly because they are acknowledged by everybody involved as important indicators of how to behave in mathematics classrooms. And collaboration is accepted as a dominant organisational format by everybody, and acknowledged as a way of ensuring that the students may progress through the tasks at greater speed than if they could not rely on assistance from one another.

The question next is how participation in KappAbel contributes as a structuring resource for the practices in Kristin's classroom. How do the projects, the problem solving tasks, and the call for collaboration restructure those practices? Does the competitive element in itself influence the ways in which the practices develop? And how is the role of these aspects – projects, problem solving, collaboration, competition - and the relationships between them transformed by their insertion into the practices developed in Kristin's classroom?

Looking through our interpretations of the transcripts in the previous subsections, it seems that in the case of Bjerkåsen and of Kristin and her students, KappAbel does play a role for how mathematics education is conducted. The collaborative efforts of the members of the mathematics department at the school contributes to and is further fuelled by a strong enthusiasm for the competition and makes participation a significant whole school commitment, also for the students. The competitive element plays a motivational role in this, as each class is keen to do better than the rest. In Kristin's classes the enthusiasm plays out in a number of different ways by supporting aspects of the current practices and inserting elements of new ones that contribute to reorganising it.

The introductory rounds of KappAbel and tasks like 'the nut of the week' (cf. section 1.2) appear to:

- support and sustain the collaborative atmosphere already present;
- contribute with inspiration on how to teach probability theory that has not had a prominent position before;
- supplement the focus on mathematical skills in a fairly narrow sense by introducing 'problem solving' tasks that require the students to read more and to apply the subject to non-mathematical situations;
- support an atmosphere in which it is accepted that there may be multiple solution methods for a given task.

For the class that did the project work there is the further potential influence that the students become involved in not only solving pre-given tasks, but in developing ways of approaching a theme with mathematical means.

However, both for the introductory rounds and for the project work, there are in some respects a certain discontinuity between the dominant modes of interaction and the ones most closely related to KappAbel. This is especially obvious with regard to the project work. For good reasons, the students would never have done the project, had they not made it to the semi-final; but because they did make it, the class spent a tremendous amount of time and energy on the project within a period of just a few weeks. The students' mathematical activity in these weeks was alien to the dominant modes engagement with the subject in the lessons observed. Also, both Kristin and her students are explicit that mathematics instruction in the project is significantly different from what they normally do.

For instance, the students explain, without being prompted, that the requirements of wording and investigating mathematical problems are alien to their ordinary classroom experiences. Doing the project, then, was a tremendous experience for the class that took part, one that they are still very enthusiastic about a year later. But it is not one that is brought to bear on the students' everyday mathematical activities. However, the very comment that the project is different indicates that participation in the project has made the students reflect on and at least implicitly challenge those modes of engagement with mathematics that normally dominate their instruction at Bjerkåsen: participation in the project does seem to have challenged the dominant beliefs about mathematics in 10.Q.

Cooperation appears to a greater extent to become part of the ways of being in mathematics classrooms at Bjerkåsen. The observations suggest that some sort of cooperation is the expectation in these classes, which is confirmed by both Kristin and the students. However, the opening in the classroom practices that allows the cooperative element in KappAbel to play a role is also the one that suggests that the intention of cooperation is to keep the speed and momentum of the whole class sessions also when the students are working independently. This means that the KappAbel intention of introducing a communal investigative enterprise through student collaboration is overruled by the expectation that cooperation is a way to ensure speedy progress with regard to the more product oriented aspects of mathematics instruction.

Also problem solving plays a role in Kristin's classroom. Like project work it is relegated to relatively isolated pockets of instruction, but for problem solving these are more frequent and shorter. In other terms, problem solving activities mainly have the character of brief add-ons in relation to the more product oriented approaches that dominate instruction. In order to shed further light on how problem solving is conceived and used, we need to look at what problem solving is taken to mean in Kristin's classroom.

In most cases Kristin and her students use the notion of problem solving to describe the students' work with tasks in which mathematics is applied in contexts beyond the subject itself. Such tasks normally require the students to read and disentangle texts in order to select information for mathematical treatment. This is exactly the main emphasis of problem solving in Kristin's classrooms, an emphasis that is directly related to the school leaving exam. It is a dominant aspect of the exam and apparently a real worry for Kristin (cf. p. 123) that the students at the exam are required to read and understand texts. She is explicit that she needs specifically to prepare the students for this. This means that the activity of preparing for the exam creates an opening in the classroom practices that allows problem solving – in the sense described above - to play a part. Worded differently, KappAbel contributes to structuring the activity of exam preparation by inserting an element of problem solving in that activity. However, the insertion of problem solving in the exam preparation also restructures the very notion of problem solving and may be seen as a contributing factor to how the understanding of the term in Kristin's classroom has developed. This is an understanding, which is in no way idiosyncratic (cf. Schoenfeld, 1992), but that leaves out competing or supplementary conceptions of the term. For instance it may be considered a sine qua non of problem solving that the solver becomes involved in activities of a non-routine character in the sense that he or she is not familiar with a standard procedure for how to solve the task in question. This includes an investigative element in the understanding of problem solving, and it invites students to come up with multiple suggestions for solutions and solution strategies, suggestions that may be discussed, compared and further developed in the process. This is not to a great extent part of the notion of problem solving as understood by Kristin and her students. In their classrooms they do in some cases discuss several ways of addressing a mathematical task. However, in the lessons we observed, the different strategies were not developed or refined by the students themselves, and the differences and relative advantages of the strategies were not made the explicit object of discussion except in the sense of one being declared the easiest or most convenient to use.

To elaborate further on this point, the investigations that may be seen as inherent in the notion of problem solving require a certain openness in the problems addressed, in the approaches adopted or in the range of possible answers. Martin and Ida comment in the student interview (cf. lines 30-47 in the previous subsection) that such openness was part of their work with KappAbel. Ida's point is that in the project you can include whatever mathematics you want in order to address the problem in question. Martin extends this, and says that it characterises the introductory rounds of KappAbel that the students are to find ways of addressing a problem, while in the project "we had to find question *and* solution, and for example *several* solutions, and more//in a way more *ways* to ask the question." It is, then, also to the students a characteristic of KappAbel that they work with problems that are open in multitude of ways. These ways of working, however, are alien to those that dominate most mathematics education in Kristin's classrooms. We observed two instances in which two ways of addressing a given task were discussed. These are the ones cited in lines 1-22 (constructing an isosceles triangle when the base line and the height are known) and 159-184 (finding the area of a map of a factory site) in the subsection on ‘Kristin and the classroom practices’. As mentioned in the analysis of the episodes, the students do not use both ways in the first episode. In the second episode they do use both ways, but they do not engage in an effort of developing them or of discussing their relative advantages and disadvantages. In general, then, they work only with a limited conception of the second of the three types of openness, i.e. the ones of openness in the wording of the problem, in the ways it may be addressed and in the range of possible answers.

It seems then, that aspects of KappAbel find their way into the dominant practices in Kristin’s classrooms at Bjerkåsen. This is so as far as the collaborative efforts are concerned and for an element of problem solving. It is also the case that KappAbel contributes with support for new mathematical topics to be dealt with, especially probability theory. However, it is a dominant aspect of KappAbel that the students become involved in collaborative efforts to investigate open tasks or problems. This is the case both in the project work and in the introductory rounds, with the main difference being the demands in terms of time and energy required and the extent to which the students are not only required to solve the problems, but also to pose new ones. These aspects of KappAbel play a part in Kristin’s classrooms only when the students participate in the competition or prepare themselves for doing it in the very immediate sense of working with previous KappAbel tasks. In this sense, KappAbel supplements rather than transforms the bulk of teaching-learning processes in Kristin’s classrooms. In spite of that, participation in KappAbel has made the students think differently about mathematics, even if the competition plays a role only in isolated pockets in the instructional sequences. This is apparent for instance from the references to the student interview repeated in this subsection. So although KappAbel has only to some extent contributed to developing the teaching-learning practices in Kristin’s classroom, it has made the students aware that mathematics may be conceived of in broader terms than merely practising pre-given procedures. Although it has not had a tremendous influence on the everyday practices, it may have supplemented the students’ beliefs about mathematics with a more processual element.

Chapter 7

Conclusions

KappAbel is a mathematics competition for school classes in the Nordic Countries. In Norway and Iceland the participating classes are grade 9; in Denmark, Finland and Sweden they are grade 8. KappAbel emphasises that mathematics is characterised not only by its products in the form of definitions, theorems, formulae, proofs, etc., but also by processes of for instance experimenting, reasoning, conjecturing, refuting, and substantiating. This is done by use of terminology like investigations, creativity, curiosity and collaboration. Also, KappAbel is meant as an inclusive competition in the sense that it is to involve whole classes and not only students, who are regarded as the most qualified in mathematics.

KappAbel has the ambition to influence mathematics teaching and learning more generally, i.e. beyond the students' and teachers' participation in the competition itself. The aim of the present study is to investigate if and how KappAbel is successful in this. The main question that we address, then, is whether participation in the KappAbel competition has the potential to influence the views of mathematics and of mathematics education of Norwegian teachers and students by influencing the practices unfolding in their mathematics classrooms. We addressed this question using both quantitative and qualitative means: Teacher Survey 1 (944), Teacher Survey 2 (15 respondents), interviews with seven teachers and five groups of students, document analysis, and a case study encompassing classroom observations.

Who participate?

The question of the potentials for change obviously relates to how many classes and teachers participate in the competition. Also, it relates to the characteristics of the ones who do participate both in terms of a range of relatively objective variables and of the teachers' mathematical and educational priorities. We therefore wanted to know whether there are systematic differences between the teachers whose classes take part in KappAbel and the ones whose classes do not.

45% of the mathematics teachers of Norwegian grade 9 classes in the academic year 2004/5 responded to the first teacher survey, TS1. These teachers come from 54% of the schools teaching lower secondary level. The proportion of the 944 responding teachers who participate in the first round of KappAbel is 35.6%. This is higher than the 25.3% of the classes that participate according to the KappAbel secretariat. It is not unexpected that the participation rate is higher among the ones who respond to the questionnaire than among other teachers.

TS1 includes questions on objective variables like gender, age, education (college or university), mathematical background and location of the school (rural or urban area). The main finding is that all but one of these variables are not systematically related to participation in KappAbel. The only exception is the teachers' mathematical background, as measured in terms the length of their studies in mathematics: The more time spent on mathematics at college or university, the greater the propensity to take part in KappAbel. The participation rate varies from 23,9% for those with weak mathematical backgrounds (credits less than 5) to more than 40% for those with a more comprehensive mathematical education (credits of more than 20). Worded differently, KappAbel appeals equally to teachers irrespective of their score on the other ”objective” variables. At least formally, then, it has the potential to influence mathematics classrooms in almost all settings irrespective of the teachers' score on these objective variables.

It was one initial hypothesis that KappAbel may be confirmative rather than formative in the sense that it is primarily used by those teachers who are already in line with the rhetoric of the reform. If this is the case, it significantly limits the potentials for change. This is so, although KappAbel may still be a significant resource for the teachers who comply with the intentions.

In order to substantiate or refute this hypothesis, we asked questions that were to indicate agreement or disagreement with the educational and mathematical priorities of KappAbel. A factor analysis of items in TS1 that were to capture the respondents' views of mathematics came up with three factors, indicating emphases on *patterns and investigations*, *structures and logical reasoning*, *tools and applications*. There is a small, but statistically significant, difference, as the teachers who are more interested in investigations have a slightly higher tendency to become involved in KappAbel than their colleagues with other priorities. A similar factor analysis of the envisaged teaching-learning environment was done. This resulted in the two factors of *collaboration* and *presentation and practice*. There were, however, no significant differences between teachers' score on these two factors and their propensity to participate in KappAbel.

Why do they (not) participate?

In general, then, the hypothesis that KappAbel is confirmative rather than formative is not substantiated by the study. In other terms, teachers with very different mathematical and educational priorities have their students take part in the competition. This means that a first ground condition of possibility for broad impact of KappAbel is met: it does not seem to scare off teachers with different mathematical or educational priorities.

But if teachers do not primarily participate because they are in line with the mathematical and educational priorities of KappAbel, then why do they participate? In TS1 the teachers were given seven options as possible reasons for their participation. 95% of the teachers whose classes do participate claim that it is in order to motivate the students, 91% that it is because the tasks are challenging and 88% that it is because KappAbel is collaborative. These are the most frequently mentioned reasons. The reason of "the theme of the year, "Mathematics and the Human Body" inspires mathematical activity" is the one mentioned the least. Only 14% if the respondents agree or agree completely that this is the case, while 27% disagree or disagree completely.

Similarly the teachers whose classes do not participate indicate their agreement or disagreement with seven possible reasons for not doing so. The most frequently mentioned reasons are that project work takes too much time in view of the learning potential and that there is no tradition of participation at the school (both 46%). Very few teachers claim not to participate because the tasks are bad (1%) or because the theme of "Mathematics and the Human Body" does not inspire mathematical activity (3%).

These findings are somewhat in line with the one above, that participation is not primarily based on an experienced compatibility between one's own views of school mathematics and those of the KappAbel. Rather, it is based on need to motivate and challenge the students and to have them collaborate. One interpretation of this is that motivation and challenge are not issues that are primarily concerned with mathematics.

Also as part of TS1, the teachers were asked to respond qualitatively to the questions raised. 253 of the teachers did so. The remarks made vary considerably. However, a substantial number of them shed further light on the reasons why teachers do or do not have their classes participate.

The most common remark is concerned with time. 70 respondents refer to lack of time as an issue in relation to KappAbel. They almost consistently state that as there are only three mathematics lessons a week in grade 9, there is too little time even without participation in activities like KappAbel. This argument makes sense only if KappAbel is considered an activity that does not contribute significantly to the students learning of mathematics. This elaborates the quantitative result mentioned above that an important reason for choosing not to participate is that the amount of time spent on project work is excessive in comparison with student learning.

The issue of time mentioned above is only related to the content and organisation of KappAbel in the sense that it indicates that KappAbel is irrelevant to the types of student learning expected in school. Another set of comments, however, are more closely related to KappAbel as such. These concern the quality of the tasks and organisation. A small number of teachers (<10) claim that KappAbel influences their everyday teaching, while a considerably larger number (25-30) state that they occasionally use ”the nut of the week”, apparently for motivational purposes or just to change their routines. In both cases it seems that KappAbel is an important source of inspiration for the teachers in question.

However there are also a number of rather more critical remarks about the tasks and organisation of KappAbel. One of these concerns the contents and the ways of working in the competition, and claim that KappAbel is not suitable for the average or below average student of mathematics. Another set of comments claim that the organisational requirements of the competition are incompatible with the situation at the school in which they work. For instance the fact that whole classes rather than individual students compete makes it difficult for small schools in rural areas to compete. This is the explicit reason why a number of the teachers at these smaller schools do not participate. A further difficulty is that the teams of four going to the semi-final are to consist of two boys and two girls. In small classes there need to be only a small imbalance between the number of boys and girls for it to become difficult to find two of each, who are sufficiently qualified.

The last major group of comments about participation and non-participation concerns information about the competition. 23 of the teachers refer to lack or insufficient information as a problem. Some argue that they do not have sufficient information about KappAbel in general, while others claim that the theme of competition is announced so late that all the plans for the year have been made and that it is awkward to reorganise in case the theme is sufficiently interesting for them to participate.

What is the potential impact of participation?

The second survey and the observations and interviews with teachers and students who do take part in KappAbel indicate that indeed the competition does have potential influence. In the qualitative part of the KappAbel study we met seven teachers and their students through a questionnaire (TS2), interviews and document analysis of project reports on “Mathematics and the Human Body”. Within the time available for the research project we only had opportunity to follow one of these teachers in a few lessons in each of three different classes. This investigation was analysed and reported as a case study.

Mathematics and mathematics teaching/ learning

In questionnaires and interviews in the qualitative part of the study, we listened to the voices of the teachers and their students. On the basis of our initial understanding of the complexity of teachers’ and students’ views of mathematics, their mathematics classroom practices, and the dynamic of their interconnectedness, we made reports on every single teacher involved in this part of the study. We structured and analysed the data from the first meeting with the six teachers (their answers in the questionnaire TS2) and from the second meeting with the teachers and their students (the transcribed dialogue in the interviews and the reports from the project “Mathematics and the Human Body”). This result is reported in the chapters 5.4 and 5.5, and here the reader will have a picture of the complexity. In the following, we want the reader to listen to the voices of a few teachers and students from six different classrooms, telling in the interview about “a really good mathematics lesson”:

> Arne (teacher): They took one hundred per cent charge of their own learning [in KappAbel]. Well, they came up with an idea, and they took it all the way through. And that goes for ordinary lessons too. …. / Well, as long as it is related to the subject or like …/ I suppose it isn’t always all that much about [the subject], but if I think there is a potential for development in it. In terms of mathematics, for them, then I just get them going.

Håvard (student): Maybe the first lesson we have when starting a new chapter. Because then ... the teacher shows us what to do, after all. And after a while it's just / then we just continue the work on our own, and we put our hands up if we need any help. But it's like first they show us the first things we need to know from the chapter.

Tina (student): Yes, breaks. Relevant to maths and somehow helping to develop our logical sense, and whatever. So it had to do with maths, but it was in a way a break from the book. This also in a way may still make us find that working with maths is fun.

Randi (teacher): You can talk about mathematics. So, if I have one way … of calculating percentages, for instance, and another in the class has another way of learning to calculate percentages and a third maybe (laughter) has learnt to calculate percentages in a third way. So we can start by asking … what is actually right or wrong.

Steinar (teacher) ... I think the lessons are mostly the same … (laughter) … It must be one of these lessons where I bring in some "mathematical nuts", where there is really thinking among the students and where I turn on … a lot of students.

Katrine (student): … That the teacher helps you, that you don't sit there with your hand up all lesson, that you get tasks suitable to … what you can … (…) Have a bit of … variation in the tasks so that it's not just the same old stuff with squared x and all that, that there are some nice tasks to work on. … Like the ones we've had in KappAbel, it's been nice to work with those tasks.

With this small mosaic of comments from teachers and students, who all participated in KappAbel's qualifying rounds and semi-final, we want to emphasise the diversity of the views of mathematics and mathematics teaching /learning and of the classroom practices in Norwegian grade 9 mathematics, as they are seen through the lenses of this study.

The case study

In the case study, we also conducted interviews with the teacher and with some of her students. The school is one with a tradition of participation in KappAbel, and the class was one that did very well in last years' competition. The data from these observations and interviews are not considered in any sense exemplary of most grade 9 classrooms in Norway, but they may be indicative of what may be hoped for under very conducive conditions.

In the case studied, the competition

- contributed to collaboration between the mathematics teachers at the school;
- occasioned that particular days' of mathematics were held as preparation for KappAbel;
- inserted elements of problem solving, primarily in terms of "word problems" in the existing structure of the classrooms practices;
- contributed further to the existing cooperative spirit, one that is primarily oriented towards ensuring speedy progress with regard to the more product oriented aspects of mathematics;
- had caused the students to make an extreme effort when doing the project for last year's semi-final.

But KappAbel-type activities have not come to dominate mathematics instruction. Investigations are not the dominant mode of mathematical activity, although elements of problem solving do play a part. Project work is not high on the agenda, or on the agenda at all, because it takes too much time: even at this school the teachers are explicit that they do not do the project, if they don't qualify for the semi-final. And classroom organisation has not changed towards greater emphasis on collaboration in large groups.

Conclusions, challenges and proposals for consideration

KappAbel has become big in Norway: almost 25% of Norwegian grade 9 classes participate to some extent. Also, there seems to be greater quantitative potential, as many of the teachers who at present decide against participating may change their position. To facilitate such a change, KappAbel may consider:

- *The time factor - timing*. At present the competition is for Norwegian grade 9. This appears to be a bad choice as the students have only 3 lessons a week while they have 4 in both grades 8 and 10. This means that a very large proportion of the teachers consider themselves and their classes under more pressure to fulfil other expectations of school mathematics in grade 9 than in the other grades. This, in turn, makes them decide against participation, in some cases in all parts of KappAbel and in others only in the project work. In grade 10, of course, the final exam plays a dominant role, and shifting KappAbel to that grade is not an option. However, one may consider moving it to grade 8. This has the dual advantages of the students having more mathematics lessons and of being further removed from the final exam after grade 10.
- *The time factor – contents*. When so many of the teachers claim that it is a problem to participate because of lack of time, it is based on the premise that participation does not qualify the students to fulfil other requirements of school mathematics. These points to a dilemma. KappAbel's priorities are closely linked to investigative mathematics. However, the emphasis on investigations is so strong that a large proportion of the teachers do not see the competition as relevant to their main obligations in relation to school mathematics. We consider this a perfectly legitimate concern on the part of the teachers. Therefore one may argue that attempts should be made to include activities that combine valuable mathematical contents with the investigative approach. This may shift the attention of the teachers from what they cannot do with their students in terms of mathematics when involved in KappAbel to the ways in which KappAbel assists their students in developing valuable mathematical knowledge and skills.
- *Procedures for choosing a theme for the project*. The theme the KappAbel project seems to have some influence on the participation. For instance ”Mathematics and the human body” appears to have had less appeal than other themes. I should be considered if a procedure for selecting the theme may be developed that to a greater extent ensure that teachers feel

comfortable with the choice made and find it relevant to the mathematical contents taught at the grade level in question. Also, such a procedure may help develop a stronger sense of ownership of KappAbel on the part of teachers.

- *Information about KappAbel*. Some teachers claim that it is necessary for them to know the theme of the project before the summer holidays, if they are to participate. The premise of the argument is that they will not participate unless they consider the theme sufficiently relevant. This is hardly an issue that has a huge impact, but it does seem a legitimate concern on the part of the teachers, and we recommend that if possible the theme should be announced in May or June rather than after the summer holidays.

KappAbel does seem to have a potential impact on the schools, teachers and students, who do take part. This is in the form of problem solving activities in ordinary classrooms and greatly increased motivation for the KappAbel activities in comparison with ordinary mathematics instruction. Also schools or teams of teachers may collaborate to develop their teaching in line with the intentions of KappAbel. However, at present the project is done only by a small minority of Norwegian grade 9 classes. Also, it seems that other influence of KappAbel is limited to fairly isolated pockets of investigative or problem solving activity that does not interfere with the dominant modes of instruction in the rest of the lessons. Most probably, the KappAbel organisers will deplore this situation. However, it does point to a way of increasing the contribution of KappAbel to mathematics teaching and learning. This is to focus on the less spectacular, but possibly more influential aspects of the competition. The ”nut of the week”, and the first two rounds are probably much more influential than the more spectacular project. We fully appreciate the need to include elements in the competition that are clearly recognisable as manifesting the identity of the competition. Also, we realise that to a large extent the project work has this role. However, one may consider whether a less ambitious format of the project may be as successful in this and at the same time more influential in most mathematics classrooms, not least if combined with shifting the competition to grade 8.

One last issue should be mentioned. KappAbel is considered an inclusive competition in the sense that it – in principle - invites all grade 9 students to take part. However, it is not entirely successful in this. First a number of teachers argue against participation on the grounds that their classes are too small or that the tasks are too difficult. These classes and students are obviously not allowed to participate. It may be expected that this is the case to greater extent in countryside schools or in areas that are relatively more hit by social problems than in other areas. In turn this suggests that the perceived imbalance between requirements and student capabilities leads to a social imbalance between those, who participate and those who do not. This problem amounts to whether the inclusive intentions of KappAbel within classes have adverse effect on the problem of inclusiveness between classes.

Also within classes, however, the intentions of inclusion may be challenged. Although the whole class is involved, there may be a tendency that a division of labour is developed in the class. This means that even if everybody is formally involved in for instance the project, the mathematical challenges are dealt with only by a limited number of students in the class, while for other students the participation is decidedly non-mathematical and deal primarily with more practical tasks related to the project. In the terminology of Christiansen and Walther (1986), learning activity is in this situation replaced by educational activity. We fully appreciate that this is often the case also in other mathematics instruction, and our comment here is not that KappAbel is more exclusive than most mathematics teaching. We just point out that if inclusion in mathematical activity is one of success criteria of KappAbel, it is not entirely successful.

No educational initiative should be expected on its own to transform the beliefs and instructional practices of students and teachers, although it may make a contribution. This is the case also for KappAbel: It has not achieved all it set out to do, but it is a significant source of inspiration for many mathematics teachers and their students. As such it has considerable potential to influence, though maybe not entirely change, the views and practices adopted in large numbers of mathematics classrooms.

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Appendices

A. Statistical analysis of Teacher Survey 1 (TS1)

B. Technical report from Teacher Survey 1 (TS1)

1.1 Questionnaire TS1

1.2 Letter to the schools (the headmaster)

1.3 Letter to the mathematics teachers

1.4 Statistics of 2004-2005 from the KappAbel secretariat

2.1 Questionnaire TS2 (electronical)

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2.3 Email on TS2 to the mathematics teachers

3.1 Interview guide (teacher)

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3.5 Letter to the students' parents (after the interview)

3.6 Letter to the teacher on the transcription (almost identical letter sent to the students)

Appendix A

Statistical Analysis of Teacher Survey 1

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Introduction

This appendix comprises the statistical analyses carried out on data from Teacher Survey 1. It is the background for the results reported in chapter 4. In Appendix B you may find the technical report on data collection and handling. The questionnaires were coded in Excel and analyses were performed in SAS version 9.1.

Data collection

The questionnaires for Teacher Survey 1 (see Appendix 1.1) were sent to the schools on December 10 2005. Answers were received from 646 out of 1189 schools (54.3%). (See Appendix B for further information.)

Variables

The baseline answers on Teacher Survey 1 are found in chapter 4.1. Table A.1 gives a list of those background variables for the teachers that are included in the analyses in this appendix.

Participation and non-response

The frame of reference for Teacher Survey 1 was all teachers having mathematics classes on level 9 in the academic year 2004/5. No central registration of the teachers existed. Hence it was not possible to obtain a list of names and addresses for the teachers. Moreover the size of the population was not known. Appendix B contains a description of the way the actual survey was carried out. There were 2851 classes on level 9 in Norway in the school year 2004/5 (Source: Norsk Skoleinformasjon) The questionnaire was answered by 944 teachers with a total of 1276 classes, corresponding to 44.8% of all classes. If we make the assumption that non response is not correlated with the number of classes the teacher had in 2004/5 we can estimate the response rate for the teachers to be 45%. The design of the survey allowed no follow-up, and for the reasons given above it was not possible to compare key variables such as gender and school location among respondents with that in the total population.

Table A.1 Background and experience of teachers

Variables	*Values*
Gender	Female
	Male
Year of birth	- 1950
	1951-60
	1961 - 70
	1971 -
Education	Teacher training college
	Bachelor
	Other
Credits in mathematics	0-4
	5-9
	10-19
	20 -24
	25-
School location	Urban
	Rural
Participation in Kappabel	Yes
	No

Table 1 in Appendix B gives an overview on the participation in Teacher Survey 1 based on classes and counties (fylker). The participation rate for classes varies moderately over counties. The lowest participation rate, 34%, was found in Troms. The county with the highest participation rate was Oppland with 68%. There is no clear geographical pattern in the participation rate, although there is a slight overrepresentation of the Eastern counties. For 54% of all schools one or more teachers had answered the questionnaire. Here again Oppland had the highest participation rate with answers from 72.7% of the schools. The county with the lowest participation rate was Finnmark with 41.3%. This might however be due to the fact that the schools in this county on the average have few classes.

How do background and experience of the teachers associate with participation in Kappabel?

In this paragraph we study the association between participation in the KappAbel competition and the background and experience of the teachers. The participation rate in KappAbel among the teachers who answered the questionnaire was 35.6%. (Table A.2). This is somewhat higher than the participation rate among classes on level 9, where the participation rate was 25.3% (720 classes out of 2851, data from the KappAbel Secretariat, see Appendix 1.5).

Table A.2 Participation rate in KappAbel compared to teacher background and experience

Background and experience	Value	Participation rate in KappAbel (per cent)	Number of Teachers[1]
All		35.6	944
Gender	Woman	38.1	391
	Man	33.9	545
Year of birth	-50	34.6	301
	51-60	40.4	228
	61-70	33.5	191
	71-	33.9	218
Education	Teacher training college	34.3	507
	Bachelor	39.8	304
	Other	29.9	117
Credits in Mathematics	0-4	23.9	92
	5-9	31.9	147
	10-19	34.9	318
	20-24	40.8	319
	25-	40.2	62
School location	Urban	39.3	389
	Rural	32.7	545

[1]Due to missing observations not all groups have the same number of observations in total.

Table A.2 shows the percentage of various background characteristics among teachers participating in the survey. This should only be considered background information, since one might expect that certain groups are overrepresented in the survey. The main interest in this report is however how participation in

KappAbel is influenced by teacher characteristics and experiences. In what follows we make the assumption that we do not have a differential non-response rate on the interaction level. This means that even if the response rate differs for instance between men and women and between participants and non-participants in KappAbel it might still be warranted to assume that the association between gender and participation will be the same for respondents and non-respondents. Technically speaking the validity of the conclusions in the following paragraphs is based on an assumption of weak or vanishing third order association between background characteristics, participation in KappAbel and response/non-response.

Table A.2 shows how the participation rate in KappAbel varies with background and experience of the teachers. The overall picture is rather homogeneous with a participation rate between 30% and 40 % in most groups.

Table A.3 Odds Ratio Estimates for logistic regression

Effect		Point Estimate	95% Wald Confidence Limits	
Gender	Man (baseline)			
	Woman	0.834	0.629	1.107
Year of birth	-50 (baseline)			
	51-60	0.818	0.564	1.187
	61-70	1.043	0.697	1.559
	71-	1.137	0.768	1.685
Education	Teacher training college (baseline)			
	Bachelor	0.860	0.613	1.171
	Andet	1.297	0.828	2.032
Credits in mathematics	0-4 (baseline)			
	5-9	0.672	0.365	1.237
	10-19	0.579	0.333	1.007
	20-24	0.445	0.258	0.766**
	25-	0.470	0.228	0.968*
School location	Urban (baseline)			
	Rural	0.763	0.579	1.006

* significant at 5%-level ** significant at 1%-level

Table A.2 gives the participation rate in the KappAbel contest compared to teacher background and experience. The results in this table might be confounded. To see how each of the background variables influences the participation rate when we control for the other background variables a logistic regression analysis was performed with *participation* in KappAbel as response variable and *gender, year of birth, education, credits in mathematics* and *school location* as background variables. The results are given in table A.3. Only *credits in mathematics* have a statistically significant association with *participation* in KappAbel, higher credits giving a higher participation rate.

Further analysis of background and experience for the teachers

In the preceding section we saw that neither *gender, age, education* nor *school location* for the teachers influenced the participation rate in the KappAbel contest in any significant way. To give more background information on association between teacher characteristics and experiences these were analyzed in a log-linear model with main effects and two factor association (see e.g. Bishop et al., 1975) The analysis was carried out by means of the SAS-procedure CATMOD. The results are shown in table A.4.

Table A.4 Association between teacher characteristics and experiences

Association	Pr > Chi- square	Significant
Gender*Year of birth	<.0001	**
Year of birth*Education	<.0001	**
Education * Credits in mathematics	0.0002	**
Education*School location	0.0483	*
Year of birth * School location	0.0569	*
Gender*Credits in mathematics	0.2107	-
Credits in mathematics * School location	0.5961	-
Gender *Education	0.6587	-
Gender*School location	0.8931	-

* borderline significant at 5%-level   ** significant at 1%-level

Age has a statistically significant association with *gender* and *education*. Only 26.5% of the teachers born before 1950 were women compared to 49% of the teachers born after 1950. The percentage of teachers with education from a teacher training college is declining with age (45.5% in the oldest group (born before 1950), 55.9% among teachers born in 1950-1970 and 64.4% for teachers born after 1970). *Education* and *credits in mathematics* are associated in the sense that teachers with a degree from a teacher training college on the average have fewer credits in mathematics (11.7 credits) compared to teachers with other education (average 12.9 credits). Finally we see that we have no statistically significant interaction between *gender* and *credits in mathematics* and between *gender* and *education.* Likewise we have no statistically significant interaction between *school location* and *credits* in mathematics and only a borderline significant association between *school location* and *age* and *education.*

Association between the teacher's school mathematical priorities and participation in KappAbel

Questions 9 and 10 in TS1 explore the teacher's school mathematical priorities as expressed in their ideas of students' activities and the purpose of theses activities in the mathematics classroom. In the questionnaire the teachers assess statements on what mathematics in schools should put *heavy* emphasis on (question 9), and what is a characteristic trait of particularly *good* mathematics teaching (spørgsmål 10). The statements are obviously interconnected. In the following section we use factor analysis as a way of reducing the dimensionality of the information and finding latent structures in the data. A separate analysis was performed for the answers to question 9 and to question 10. The notes to the analysis of question 9 also pertains to the analysis of question 10.

"Mathematics in schools should put *heavy* emphasis on"

A factor analysis of the 11 items in question 9 with three factors and varimax-rotation (see e.g. Mardia, K. & B., 1979) was performed and gave the factor pattern in table A.5[1] . Values > 45 are highlighted, since a high score indicates that the item gives a high contribution to the factor in question.

Table A.6 summarises the results from table A.5 giving the main content of each of the three factors. The first factor is seen to represent attitudes connected with the discovery of rules and patterns, with investigations and formulation of new mathematical questions and is termed *patterns and investigations*. The second factor is primarily connected with logic and structure and is termed *structures and logical reasoning.* The third factor is connected with practical everyday problems, formulas and methods and relevance to future work, hence it represents a *tools and applications* dimension. Almost all teachers (97%) answer that item 9.11 "letting students practice the basic skills of the subject" is

[1] In the factor analysis missing values were imputed with a value corresponding to the mean value of the non missing values.

important. Hence this item does not correlate specifically with any of the three dimensions. Neither do the item 9.5 “letting students learn some of the classic mathematical proofs” but this is not because all teachers find it important.

Tabel A.5. Factor scores for statements on what mathematics in schools should put *heavy* emphasis on (question 9 in TS1)

Item (Statement)		Score Factor 1	 Factor 2	 Factor 3
9.2	giving students the opportunity to discover and generalize about mathematical connections, rules and patterns	**66**	10	20
9.4	letting students experiment in order to find and describe solutions to mathematical problems	**64**	12	21
9.9	letting students explore hypotheses on mathematical problems – their own as well as those of others	**60**	44	14
9.7	encouraging students to ask innovative mathematical questions	**49**	38	16
9.10	letting students work on logical mathematical reasoning	24	**62**	14
9.8	Presenting mathematics to the students as logical connections and structures	11	**58**	16
9.6	letting students experience how mathematics can help them solve practical everyday problems	31	21	**49**
9.3	letting students learn mathematical formulas and methods applicable to real problems	13	25	**47**
9.1	letting students learn mathematics relevant to their future careers.	15	4	**46**
9.11	letting students practise the basic skills of the subject	3	35	34
9.5	letting students learn some of the classic mathematical proofs	23	29	13

To analyse differences between the teachers who did or did not participate in the KappAbel contest the median factor score over the whole population was calculated for each factor (dimension) and the percentage of teachers with factor scores above the median was calculated for the participants and for the non-participants and the percentages were compared[2].

Table A.6 Factor content: The three value dimensions constructed in the factor analysis of TS1 question 9

Dimension	Name/value	Key items
Factor 1	Patterns and investigations	Discover rules and patterns, experiment, explore, ask
Factor2	Structures and logical reasoning	Logical reasoning, logical connections and Structures
Factor3	Tools and applications	Practical everyday problems, formula and methods, relevant to future work

Table A.7 shows a small but statistically significant difference for the dimension designating patterns and investigations, in the sense that teachers participating in KappAbel have a slightly higher score on this dimension. For the two other dimensions there is no association between score and participation in KappAbel.

Table A.7 Percentage of teachers with score above the median for each factor in TS1 question 9. Separately for participants and non-participants in KappAbel

.

Dimension (Factor)	Participation in KappAbel	
	Yes	No
Patterns and investigations*	55.0%	47.4%
Structures and logical reasoning	50.0%	49.8%
Tools and applications	50.0%	50.0%

* significant with p=0.02

[2] Since the dividing point for each factor is chosen as median for the whole population the percentages shows deviation from the common median.

A supplementary analysis of association between participation in KappAbel and answers on the single items in question 9 gave no statistically significant results (p-values for the chi^2-test varied between 0.11 and 0.81.)

"What is a characteristic trait of particularly *good* mathematics teaching "

A factor analysis of the 7 items in question 10 with two factors and varimax-rotation was performed and gave the factor pattern in table A.5. Values > 45 are highlighted, since a high score indicates that the item gives a high contribution to the factor in question.

Table A.8. Factor scores for statements on characteristic traits of particularly *good* mathematics teaching that (question 10 in TS1)

Item (Statement)		Score Factor 1	Score Factor 2
10.2	the students discuss each others' suggestions and methods.	**74**	-1
10.3	the students discover the world of mathematics together.	**68**	-4
10.6	the students learn to co-operate in solving mathematical problems.	**67**	2
10.5	the students explore and discuss before the teacher goes through the topic on the blackboard.	**63**	-2
10.7	the students work, among other things, on projects that include subjects in addition to mathematics	**45**	-10
10.1	the teacher spends a good deal of time on thoroughly explaining to the students the methods and concepts of the subject.	-10	**54**
10.4	the students achieve routine proficiency in solving mathematical exercises.	4	**53**

Table A.9 summarises the results from table A.8 giving the main content of each of the two factors. The first factor is seen to represent attitudes connected with discussion and co-operation between students, is termed a *collaboration* dimension. The second factor is primarily connected with the teacher explaining the methods and concepts of the subject and the students achieving routine proficiency in solving mathematical exercises. It is termed a *presentation and practice* dimension.

To analyse differences between the teachers that did or did not participate in the KappAbel contest the median factor score over the whole population was calculated for each factor (dimension) and the percentage of teachers with factor scores above the median was calculated for the participants and for the non-participants and the percentages were compared. (Table A.10)

Table A.9 Factor content: The two value dimensions constructed in the factor analysis of TS1 question 10

Dimension	Name/value	Key items
Factor 1	Collaboration	Discussion, co-operation
Factor 2	Presentation and Practice	Explain methods, routine proficiency

Table A.10 Percentage of teachers with score above the median for each factor in TS1 question 9. Separately for participant and non-participants in KappAbel

Dimension (Factor)	Participation in KappAbel	
	Yes	No
Collaboration	53.4%	47.9%
Presentation and Practice	48.8%	50.4%

There is no statistically significant association between score and participation in KappAbel for any of the dimensions. A supplementary analysis of association between participation in KappAbel and answers on the single items in question 10 gave no statistically significant results (p-values for the chi^2-test varied between 0.15 and 0.95.)

The results in this section confirm the finding that there is no big difference between teachers participating and not participating in KappAbel .

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Appendix B

Technical report from Teacher Survey 1

Population

We included all teachers of mathematics in grade 9 in the academic year 2004/05 in Teacher Survey 1 (TS1). One of the reasons for this was that no statistical information on this specific group of mathematics teachers (e.g. age, gender or educational background) was available in Norway. From the statistical information from KappAbel, we also knew that the percentage of school participation varied very much from county to county e.g. from 13% to 55% in the academic year 2004/05 (see appendix 1.4). We didn't know the size of the population but had a qualified guess on the basis of available information from Statistics Norway on the number of grade 9 classes. Thus, it was necessary to buy the information (schools with 9th grade, county, addresses, number of grade 9 classes at each school) at Norsk Skoleinformasjon (Norwegian School Information http://ped.lex.no).

According to this, the corrected number of relevant schools was 1189 and the number of classes at 9th grade was 2851 (see table 1). As we expected that quite a few teachers taught mathematics in more than one of theses classes, the estimated number of teachers in the population was between 1500 and 2500. (See more about the population in chapter 4.)

Sending out the questionnaire

From Norsk Skoleinformation we received a register with addresses of schools teaching lower secondary students[1]. We also obtained information about the number of grade 9 classes in each school in the academic year 2004/05. We used this information to code the envelopes and send out the letters to the schools. (A summary of this information is presented in table 1).

---

[1] Some of the schools only teach students at this level, while others also teach at primary level. In the following we shall refer to all of them as lower secondary schools.

Table 1 Number of schools teaching lower secondary students and number of questionnaires

County	Number of schools on the mailing list[i]	Number of schools from where we received an answer	Number of question-naires send out (classes)[ii]	Number of received question-naires (teachers)	Received questionnaires (classes)[iii]	
					number	per cent
Østfold	44	29	138	53	75	54%
Akerhus	81	52	283	82	140	50%
Oslo	62	36	214	57	85	40%
Hedmark	40	24	105	37	52	50%
Oppland	44	32	106	52	72	68%
Buskerud	53	28	145	50	72	50%
Vestfold	38	24	129	52	67	52%
Telemark	42	23	102	39	50	49%
Aust-Agder	27	13	63	18	24	38%
Vest-Agder	36	21	100	33	40	40%
Rogaland	98	55	256	84	109	43%
Hordaland	100	59	269	81	108	40%
Sogn & Fjordane	51	29	87	39	47	54%
More & Romsdal	81	45	172	61	75	44%
Sør-Trøndelag	65	32	155	38	60	39%
Nord-Trøndelag	45	26	94	31	41	44%
Nordland	137	57	213	68	82	39%
Troms	81	34	133	41	45	34%
Finnmark	63	26	86	27	31	36%
Svalbard	1	1	1	1	1	100%
In total	**1189**	**646** 54 % [iv]	**2851**	**944**	**1276** [v]	45%

Source: Norsk Skoleinformasjon, PEDLEX, Oslo, http://ped.lex.no

The questionnaire was sent on the 10th of December 2004 – in between the two qualifying rounds of KappAbel – to 1189 lower secondary schools (see the questionnaire in appendix 1.1). Each envelope, addressed to the headmaster (see the letter in appendix 1.2) contained a number of letters addressed to the mathematics teachers at his/her school (see the letter in appendix 1.3) with a questionnaire and a stamped and addressed envelope for their reply. At the same time an electronic version of the questionnaire was put on the web-site. The schools received between one and seven questionnaires depending on the information from Norsk Skoleinformation on the number of grade 9 classes.

Each reply envelope had a seven digits identification number with information on county, school, entry, e.g. 15-0814-2. The county number (15 in the example) was also printed manually on the front page of each questionnaire. This made it possible to locate the replies and count how many schools that had responded from each county. This also made it possible to identify the schools if we decided to send a reminder, which we did not do.

In Teacher Survey 1 we kept *anonymity*, meaning that no identifying information was attached to the data, and thus no one, not even the researchers, could trace back to the individual providing them. (See more about ethics in chapter 3, p. 60).

Handling the data

We received 850 reply envelopes and 94 electronic replies in total 944 replies – from 646 schools (see table 1). Considering the anonymity, envelopes and questionnaires were separated before coding. The questionnaires were coded and statistical analysis was carried out (see appendix A and chapter 4.1). 253 of the teachers submitted qualitative remarks of some kind in the questionnaire. These were transcribed and analyzed (see chapter 4.2).

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[i] Number of schools indicates the number of schools to whom we sent letters. The number is smaller than the number of schools in the register from Norsk Skole-informations. There are various reasons for this: A series of schools didn’t have any grade 9 classes in 2004/05, and two schools from Rogaland and Finnmark informed us on the telephone that they did not have any classes in secondary school in 04/05. Finally, letters were returned from two schools in Østfold and Nordland that had three classes in total.

[ii] Number of grade 9 classes is calculated from the information from Norsk Skoleinformation on the number of schools and the number of classes at each and reduced with the number of classes not established, cf. information in note i.

[iii] The number is based on the number of classes stated by the teachers in the questionnaire.

[iv] The per cent is a minimum because the number of relevant schools could be smaller than the number of schools on the mailing list, cf. note i. There might be more schools without grade 9 classes in 04/05 than the schools that have given this information.

[v] 13 teachers did not answer the question on number of classes at grade 9 in 04/05.

This questionnaire is part of the Nordic research project *Changing attitudes and practices?* The research project relates to the mathematics competition KappAbel.

The questionnaire has been sent to all Norwegian schools offering lower secondary education, and is to be distributed among teachers involved in Year 9 (14-15-year-olds) mathematics courses during the school year of 2004/2005.

Completed questionnaires will be treated anonomously.

We kindly ask you to return the completed questionnaire in the enclosed envelope, or to submit your answers electronically.

Thank you for your help!

PTO

A. Background and experience

1. Gender: female: ☐ male: ☐

2. Year of birth: 19 ☐☐

3.1 Education:

teacher training college ☐ 2-years ☐ 3-years ☐ 4-years ☐

Bachelor’s degree ☐ Which subjects? ____________________

other courses/education ☐ ___________________________________

3.2 The college level maths component of your education corresponds to approximately:

None ☐
1/4 year (5 credits) ☐
1/2 year (10 credits) ☐
1 year (20 credits) ☐
Course on the teaching theories of mathematics (didactics) ☐

Other: ____

4. I have taught maths for _____ years.

5. What other subjects do you teach/are you qualified to teach? _______________________

____

6.1 This school year I am teaching ____________ weekly lessons of maths.

6.2 How many Year 9 mathematics courses/classes are you teaching this school year? ____

7. My school is located in

an urban area ☐ a rural area ☐

Please assess the statements below by circling a number: -2 means Strongly Disagree, -1 means Disagree, 0 means Undecided, 1 means Agree, and 2 means Strongly Agree.

8.

How do you perceive of yourself?	Disagree - Agree
8.1 I am a mathematician	-2 –1 0 1 2
8.2 I am a teacher	-2 –1 0 1 2
8.3 I am a mathematics teacher	-2 –1 0 1 2

PTO

B. On teaching and learning mathematics

9.

Mathematics in schools should put *heavy* emphasis on		Disagree - Agree
9.1	letting students learn mathematics relevant to their future work	-2 –1 0 1 2
9.2	giving students the opportunity to discover and generalise mathematical relations, rules and patterns	-2 –1 0 1 2
9.3	letting students learn mathematical formulas and methods applicable to real problems	-2 –1 0 1 2
9.4	letting students experiment in order to find and describe solutions to mathematical problems	-2 –1 0 1 2
9.5	letting students learn some of the classic mathematical proofs	-2 –1 0 1 2
9.6	letting students experience how mathematics can help them solve practical everyday problems	-2 –1 0 1 2
9.7	encouraging students to ask innovative mathematical questions	-2 –1 0 1 2
9.8	presenting the students with the logical relations and structures of mathematics	-2 –1 0 1 2
9.9	letting students explore hypotheses on mathematical problems – their own as well as those of others	-2 –1 0 1 2
9.10	letting students work on logical mathematical reasoning	-2 –1 0 1 2
9.11	letting students practise the basic skills of the subject	-2 –1 0 1 2

PTO

10.

It is a characteristic of particularly *good* mathematics teaching that		Disagree – Agree
10.1	the teacher spends a good deal of time on thoroughly explaining to the students the methods and concepts of the subject.	-2 –1 0 1 2
10.2	the students discuss each others' suggestions and methods.	-2 –1 0 1 2
10.3	the students discover the world of mathematics together.	-2 –1 0 1 2
10.4	the students achieve routine proficiency in solving mathematical exercises.	-2 –1 0 1 2
10.5	the students explore and discuss before the teacher goes through the topic on the blackboard.	-2 –1 0 1 2
10.6	the students learn to co-operate in solving mathematical problems.	-2 –1 0 1 2
10.7	the students work, among other things, on projects that include subjects in addition to mathematics.	-2 –1 0 1 2

C. On KappAbel

11. Are you teaching one or more classes that participate in KappAbel this year?

yes: ☐ (Go to question 12) no: ☐ (Go to question 15)

12. How many of your classes are participating? ________________

13. Will your class(es) be doing a project on Mathematics and the Human Body even if they fail to make it to the semi-final of KappAbel?

yes: ☐ no: ☐

PTO

14.

My class(es) is(are) participating in KappAbel		Disagree - Agree
14.1	in order to stimulate student interest in mathematics	-2 –1 0 1 2
14.2	because the students wanted to participate	-2 –1 0 1 2
14.3	because the problems represent a challenge	-2 –1 0 1 2
14.4	because my school has a tradition of participating	-2 –1 0 1 2
14.5	because the competition offers opportunities for collaboration	-2 –1 0 1 2
14.6	because this year's topic on Mathematics and the Human Body is a source of inspiration for engaging in mathematical activities	-2 –1 0 1 2
14.7	because the competition format may offer an opportunity for some students to take a more active part	-2 –1 0 1 2

Go to question 16

15.

My class(es) is(are) not participating in KappAbel		Disagree - Agree
15.1	because competitions are generally a bad idea	-2 –1 0 1 2
15.2	because the students do not want to participate	-2 –1 0 1 2
15.3	because the problems are not good enough	-2 –1 0 1 2
15.4	because my school does not have a tradition of participating	-2 –1 0 1 2
15.5	because the project is too time-consuming in relation to what the students get out of it	-2 –1 0 1 2
15.6	because this year's topic of Mathematics and the Human Body does not represent a source of inspiration for engaging in mathematical activities	-2 –1 0 1 2
15.7	because the competition format may serve to isolate some students even further	-2 –1 0 1 2

PTO

16.

It is a positive aspect of KappAbel that		Disagree – Agree
16.1	the entire class collaborates and submits a common solution	-2 –1 0 1 2
16.2	the class undertakes a project based on a given topic	-2 –1 0 1 2
16.3	there are two boys and two girls on each finalist team	-2 –1 0 1 2

17.
Has any previous class of yours participated in KappAbel?

yes: ☐ Did your class undertake a project on the topic set by KappAbel? yes: ☐ no: ☐

no: ☐

Optional supplementary comments:

____

____

____

____

____

Please submit the completed form in the enclosed envelope. (Postage will be paid by the addressee.)

Thank you for your help!

Nasjonalt Senter for Matematikk i Opplæringen

Til rektor

Trondheim, 8. desember 2004

Spørreundersøkelse blant alle matematikklærere på 9. trinn

Vi ber om at vedlagte spørreskjema deles ut til lærere som underviser i matematikk på 9. trinn, skoleåret 2004/2005

Spørreskjemaet inngår i det nordiske forskningsprosjektet ”*Endring i holdninger og praksis?*” og er sendt ut til alle norske skoler med ungdomstrinn. Prosjektet er initiert av Nordisk Kontakt Komité for den 10. internasjonale kongress om matematikkundervisning (ICME-10) og finansiert av Nordisk Ministerråd og Nasjonalt Senter for Matematikk i Opplæringen.

Formålet med forskningsprosjektet er å lage en evaluering av matematikkonkurransen KappAbel. Det er derfor viktig at vi mottar besvarte skjema både fra skoler som deltar i KappAbel og fra skoler som *ikke* deltar.

Svarfristen er mandag **den 20. desember** 2004.

Vi håper at skolen vil støtte opp om undersøkelsen.
På forhånd takk!

Med vennlig hilsen

Tine Wedege
Prosjektleder

Postadresse
Høgskoleringen 5
N-7491 Trondheim

Besøksadresse
Realfagsbygget
www.matematikksenteret.no

Telefon +47 73 55 11 42
Telefaks +47 73 55 11 40

Appendix 1.3

Nasjonalt Senter for
Matematikk i Opplæringen

Til matematikklæreren

Trondheim, 8. desember 2004

Vedlagte spørreskjema inngår i det nordiske forskningsprosjektet ”*Endring i holdninger og praksis?*”. Skjemaet er sendt ut til alle norske skoler med ungdomstrinn, til fordeling blant matematikklærere som underviser i matematikk på 9. trinn, skoleåret 2004/2005.

Prosjektet er initiert av Nordisk Kontakt Komité for ICME-10 og finansiert av Nordisk Ministerråd og Nasjonalt Senter for Matematikk i Opplæringen.

Formålet med forskningsprosjektet er å lage en evaluering av matematikkonkurransen KappAbel. Det er derfor viktig at vi mottar besvarte skjemaer både fra lærere som deltar i KappAbel og fra lærere som *ikke* deltar/kjenner til denne konkurransen.

Svarene på spørreskjemaet er anonyme når de blir behandlet.

Vi håper at du vil bruke ca. 10 minutter til å besvare skjemaet, og ber deg returnere skjemaet i medfølgende svarkonvolutt. Mottaker betaler porto. Det er også mulig å besvare skjemaet elektronisk på http://www.matematikksenteret.no/hogp/

Hvis du fyller ut det elektroniske skjemaet, skal du bruke skjemanummeret på svarkonvolutten som passord.

Svarfristen er mandag **den 20. desember** 2004.
På forhånd takk!

Med vennlig hilsen

Tine Wedege
Prosjektleder

Om anonymiteten i spørreskjemaundersøkelsen:

- *På svarkonvolutten* er en 7-sifret kode (skjemanummeret). Den angir fylke (siffer 1-2), skole (siffer 3-6) og lærer (siffer 7). Koden blir bare brukt til å notere hvor skjemaet er sendt fra (fylke og skole).
- Kode og skjema atskilles før data i skjemaet blir behandlet.
- *På skjemaet* står en 2-sifret kode som angir fylket skolen din ligger i. Denne opplysningen brukes ved databehandlingen.

Postadresse	Besøksadresse	Telefon	+47 73 55 11 42
Høgskoleringen 5	Realfagsbygget	Telefaks	+47 73 55 11 40
N-7491 Trondheim	www.matematikksenteret.no		

KappAbel 2004/2005 - Deltakerstatistikk

	Påmeldt				Levert svar 1. runde				Levert svar 2. runde				Frafall runde 1 - runde 2	
Fylke	**Kl.**	**Sk.**	**Sk. tot.**	**% delt.**	**Kl.**	**Sk.**	**Sk. tot.**	**% delt.**	**Kl.**	**Sk.**	**Sk. tot.**	**% delt.**	**Klasser (%)**	**Skoler (%)**
Østfold	55	19	56	33,93	30	13	56	23,21	26	10	56	17,86	13,33	23,08
Akershus	101	29	91	31,87	83	27	91	29,67	74	23	91	25,27	10,84	14,81
Oslo	57	18	65	27,69	40	13	65	20,00	35	11	65	16,92	12,50	15,38
Hedmark	54	17	43	39,53	41	15	43	34,88	23	10	43	23,26	43,90	33,33
Oppland	35	16	47	34,04	34	16	47	34,04	22	12	47	25,53	35,29	25,00
Buskerud	34	18	55	32,73	27	15	55	27,27	24	14	55	25,45	11,11	6,67
Vestfold	41	12	39	30,77	39	12	39	30,77	24	7	39	17,95	38,46	41,67
Telemark	23	9	44	20,45	19	7	44	15,91	14	5	44	11,36	26,32	28,57
Aust-Agder	40	16	29	55,17	40	16	29	55,17	36	15	29	51,72	10,00	6,25
Vest-Agder	42	14	40	35,00	42	14	40	35,00	34	11	40	27,50	19,05	21,43
Rogaland	72	29	94	30,85	67	28	94	29,79	33	19	94	20,21	50,75	32,14
Hordaland	58	25	103	24,27	47	22	103	21,36	27	14	103	13,59	42,55	36,36
Sogn og Fjordane	25	19	52	36,54	23	17	52	32,69	21	15	52	28,85	8,70	11,76
Møre og Romsdal	33	15	89	16,85	24	12	89	13,48	20	10	89	11,24	16,67	16,67
Sør-Trøndelag	59	20	68	29,41	45	15	68	22,06	30	12	68	17,65	33,33	20,00
Nord-Trøndelag	41	17	46	36,96	32	12	46	26,09	27	12	46	26,09	15,63	0,00
Nordland	46	25	144	17,36	34	19	144	13,19	29	17	144	11,81	14,71	10,53
Troms	38	21	82	25,61	32	15	82	18,29	26	11	82	13,41	18,75	26,67
Finnmark	22	12	63	19,05	20	11	63	17,46	10	5	63	7,94	50,00	54,55
Svalbard	1	1	1	100,00	1	1	1	100,00	1	1	1	100,00	0,00	0,00
Totalt i Norge	**877**	**352**	**1.251**	**28,14**	**720**	**300**	**1.251**	**23,98**	**536**	**234**	**1.251**	**18,71**	**25,56**	**22,00**
													74,44	78,00
Norske skoler i utlandet	3	2	2	100,00	3	2	2	100,00	2	1	2	50,00	33,33	50,00
Norske skoler totalt	**880**	**354**	**1.253**		**723**	**302**	**1.253**		**538**	**235**	**1.253**			
Danmark	56	31			50	30	31	96,77	46	28	31	90,32	8,00	6,67
Finland	37	14			29	11	14	78,57	21	11	14	78,57	27,59	0,00
Island	45	25			37	22	25	88,00			25	0,00	100,00	100,00
Sverige	75	37			37	20	37	54,05	27	18	37	48,65	27,03	10,00
N+DK+FIN+IS+S	**1.093**	**461**			**876**	**385**			**632**	**292**			**27,85**	**24,16**
													72,15	75,84
Ikke-nordiske deltakere	5	4	4	100,00	5	4	4	100,00	4	4	4	100,00	20,00	0,00
Samlet deltakelse	**1.098**	**465**			**881**	**389**			**636**	**296**				

Pr. 04.02.05

Appendix 1.4

This questionnaire is part of the Nordic research project *Changing attitudes and practices?* that is based on the mathematics competition KappAbel.

The questionnaire has been sent out to mathematics teachers who participated with one or several groups in the second round of KappAbel during the school year of 2004/2005, and who have confirmed that they will be continuing with the project on “Mathematics and the Human Body”.

Your completed questionnaire will be treated anonymously unless you submit your name on the final page.

Kindly use the enclosed envelope to submit your completed questionnaire (postage will be paid by the addressee); or use the form sent to you via e-mail to submit your answers electronically.

Thank you for your help!

PTO

A. Background and experience

1. Gender: female: ☐ male: ☐

2. Year of birth: 19 ☐☐

3.1 Education:
teacher training college ☐ Bachelor's degree ☐
other courses/education ☐ Please specify _______________

3.2 The college/university level maths component of your education corresponds to approximately:
None ☐
1/4 year (5 credits) ☐
1/2 year (10 credits) ☐
1 year (20 credits) ☐
more than 1 year (>20 credits) ☐
Course on the teaching theories of mathematics (didactics) ☐

4. I have taught/been teaching mathematics for _____ years.

5. What other subjects do you teach/are you qualified to teach?________________________

____

6.1 This school year I am teaching ___________ weekly lessons of maths.

6.2 How many Year 9 mathematics courses/classes are you teaching this school year?______

6.3 Has any previous class of yours participated in KappAbel? yes: ☐ no: ☐

7. My school is located in an urban area ☐ a rural area ☐

Please assess the statements below by circling a number: -2 means Strongly Disagree, -1 means Disagree, 0 means Undecided, 1 means Agree, and 2 means Strongly Agree.

8.

How do you perceive of yourself?		Disagree - Agree
8.1	I am a mathematician	-2 –1 0 1 2
8.2	I am a teacher	-2 –1 0 1 2
8.3	I am a mathematics teacher	-2 –1 0 1 2

B. On teaching and learning mathematics

9. Please assign a total of 40 points to the following four statements:

Mathematics as a school subject should put *heavy* emphasis on		**Points**
9.1	letting students learn mathematics applicable to their everyday lives.	
9.2	letting students learn how to be creative and inquisitive in mathematics.	
9.3	letting students learn logical reasoning and how to see the inter-connectedness of mathematics.	
9.4	letting students learn how to master basic skills.	
	In total	**40**

10. Please assign a total of 30 points to the following three statements:

It is a characteristic trait of particularly *good* mathematics teaching that		**Points**
10.1	the students discuss, co-operate, and conduct mathematical investigations.	
10.2	the students attain routine proficiency in the subject's methods and skills.	
10.3	The students work on projects that include subjects additional to mathmatics (inter-disciplinary projects).	
	In total	**30**

C. A good teacher

11. Think of a colleague whom you consider a good mathematics teacher: What is it that he or she masters, unlike other colleagues whom you consider less good at teaching the subject?
Please assess the statements below by circling a number: -2 means Strongly Disagree, -1 means Disagree, 0 means Undecided, 1 means Agree, and 2 means Strongly Agree.

A good mathematics teacher is *particularly* skilled at		Disagree - Agree
11.1	giving short and concise answers and explanations to students' questions.	-2 –1 0 1 2
11.2	using a variety of teaching methods.	-2 –1 0 1 2
11.3	creating an atmosphere where the students discuss each others' suggestions and methods.	-2 –1 0 1 2
11.4	getting the students started at exploring mathematical problems on their own.	-2 –1 0 1 2
11.5	explaining the methods to be used by the students, so that they do not spend too much time on trial-and-error.	-2 –1 0 1 2
11.6	engaging in and understanding difficult mathematics.	-2 –1 0 1 2
11.7	linking mathematics to the students' everyday lives and to the the world around us.	-2 –1 0 1 2

Your comments: ____

____

____

D. Your own teaching

12.1 Think back to your last really successful mathematics lesson. What traits characterise a good class in mathematics?

Key words: ____

____

12.2 Think back to the last mathematics lesson where things did not work out the way you had intended. What traits characterise a poor class in mathematics?

Key words: ____

____

12.3 What do you consider the most difficult aspect(s) of teaching mathematics?

Key words: ____

____

12.4 What do you consider the aspect(s) of teaching mathematics that are most fun?

Key words: ____

____

12.5 Do you generally prefer teaching mathematics to teaching other subjects?

Key words: ____

____

E. A successful student

13. Think of a student who is good at mathematics – what is it that he or she masters, unlike other students who are less successful in the subject?

Please assess the statements below by circling a number: -2 means Strongly Disagree, -1 means Disagree, 0 means Undecided, 1 means Agree, and 2 means Strongly Agree.

A successful mathematics student is *particularly* good at		Disagree – Agree
13.1	solving tasks quickly and correctly	-2 –1 0 1 2
13.2	finding their own way of solving a problem.	-2 –1 0 1 2
13.3	understanding when mathematics can be applied, and managing to apply the subject correctly.	-2 –1 0 1 2
13.4	remembering rules and facts (e.g. the tables).	-2 –1 0 1 2

PTO

The successful mathematics student is *particularly* good at		Disagree – Agree
13.5	asking new mathematical questions on the basis of the answers they have already found.	-2 –1 0 1 2
13.6	explaining their reasoning using everyday language and mathematical terms.	-2 –1 0 1 2

Your comments: ____

____

____

F. On KappAbel

14. How many of your classes are participating? ________________

15.

My class(es) is(are) participating in KappAbel		Disagree – Agree
15.1	in order to stimulate student interest in mathematics.	-2 –1 0 1 2
15.2	because the students wanted to participate.	-2 –1 0 1 2
15.3	because the tasks are challenging.	-2 –1 0 1 2
15.4	because my school has a tradition of participating.	-2 –1 0 1 2
15.5	because the competition offers opportunities for co-operation.	-2 –1 0 1 2
15.6	because this year’s topic on Mathematics and the Human Body is a source of inspiration for engaging in mathematical activities.	-2 –1 0 1 2
15.7	because the competition format may offer an opportunity for some students to take a more active part.	-2 –1 0 1 2
15.8	because the students like to compete.	-2 –1 0 1 2
15.9	because the students are motivated by the prize money.	-2 –1 0 1 2

15.10 Do you have other reasons for participating?

____

____

16. Please tick

On your participation in KappAbel, round 1:		Yes	No	Un-deci
16.1	The students prepared to take part in the first round, e.g. by solving tasks from previous years.			
16.2	The students continued to work on solving tasks after they received the result.			
16.3	The students were organised into groups, and each group worked on solving all the tasks.			
16.4	The students were organised into groups. Each group worked on solving some of the tasks, according to a distribution of tasks agreed upon in advance.			
16.5	The students worked on solving the tasks individually or in pairs.			

16.6 Do you wish to add anything in particular about your participation in rounds 1 and 2?

____

____

____

____

G. On co-operation

17. KappAbel puts emphasis on co-operation among the students.

17.1 Do you consider this emphasis a good idea? yes: ☐ no: ☐ undecided: ☐

17. 2 Why / why not? ____

____

____

18. In a number of interviews, teachers have been asked about their views on co-operation between the students as part of mathematics teaching. Please offer comments (in key words) to any statements you find important or provocative.

18.1 **Oscar:** *Co-operation in mathematics should only be used for small and concisely formulated tasks. Otherwise, talk among the students more often than not tends to be about anything but the subject matter.*

Key words: ______________________________

____

____

18.2 **Janne:** *The students need to learn how to co-operate, but it has little impact on their competence in mathematics.*

Key words: ______________________________

____

____

18.3 **Ole:** *Students learn mathematics by putting their thoughts into words. They should therefore co-operate and discuss with each other.*

Key words: ______________________________

____

____

18.4 **Katrine:** *The students will not learn any mathematics unless they spend a good deal of time delving into the topic on their own. Co-operation can easily be exaggerated in mathematics.*

Key words: ______________________________

____

____

18.5 **Arne:** *The students can challenge each other's understanding when working together. It is therefore important that they co-operate when learning mathematics.*

Key words: ______________________________

____

____

18.6 **Marit:** *Co-operation works best when a teacher is in charge. Therefore it should almost exclusively take place when the class as a whole are solving tasks together, with a student or myself at the blackboard.*

Key words: ____

____

____

19. Think about your work with your grade 9. Tick the appropriate column for each line of the form to indicate how often you use the method described:

		Almost every lesson	Once or twice a week	1 to 3 times a month	More rarely
19.1	The students work individually				
19.2	The students work in pairs				
19.3	The students work in groups of three or more				
19.4	The students work together as a class with the teacher or a student at the blackboard.				

20.1 What do you consider the greatest strengths and weaknesses associated with using co-operation in mathematics?

Key words: ____

____

____

20.2 What do you consider the greatest strengths and weaknesses associated with individual work with mathematics?

Key words: ____

____

____

H. On project work

21. One of the basic ideas of KappAbel is that the students should be involved in a project on mathematics within an interdisciplinary or practical framework.

21.1 Do you think this is a good idea? yes: ☐ no: ☐ undecided: ☐

21.2 Why /why not? ____

____

____

____

22. Is it a good idea to work on mathematical projects, even in contexts other than KappAbel?

yes: ☐ (Go to question 23) no: ☐ (Go to question 24) undecided: ☐ (Go to question 25)

Please assess the statements below by circling a number: -2 means Strongly Disagree, -1 means Disagree, 0 means Undecided, 1 means Agree, and 2 means Strongly Agree.

23.

Working on mathematical projects is a good thing		Disagree – Agree
23.1	because the students get to use mathematical skills they have already acquired.	-2 –1 0 1 2
23.2	because it offers a good way for the students to learn new mathematical skills.	-2 –1 0 1 2
23.3	because this method has a very motivating effect on the students.	-2 –1 0 1 2
23.4	because the students can take part in deciding what to work on and how to work on it.	-2 –1 0 1 2
23.5	because the students may change their attitudes to, and their views of, what mathematics is about.	-2 –1 0 1 2
23.6	because the students get ample opportunity for talking about mathematics.	-2 –1 0 1 2
23.7	because every student in the class can take part according to their own ability, performing a role and taking on a responsibility suitable to their level of proficiency.	-2 –1 0 1 2

It is a good idea to work on mathematical projects,		Disagree - Agree
23.8	but the students will be given very precise mathematical tasks to work on in their projects.	-2 –1 0 1 2
23.9	but such projects should ideally be carried out in collaboration with other subjects.	-2 –1 0 1 2

Your comments: ____

____

____

____

24.

Projects should not be part of the ordinary mathematics course. *If* projects are undertaken		Disagree – Agree
24.1	the students do not get sufficient opportunity to immerse themselves in mathematical terminology.	-2 –1 0 1 2
24.2	the students do not get sufficient opportunity to practise their mathematical skills.	-2 –1 0 1 2
24.3	the students do not engage much in the *mathematics* involved, even if they think the project is fun.	-2 –1 0 1 2
24.4	they give the students a false impression of what constitutes true mathematics.	-2 –1 0 1 2
24.5	other and better ways of working with mathematics in the students' everyday lives have to be omitted.	-2 –1 0 1 2
24.6	the students will talk about anything but mathematics.	-2 –1 0 1 2
24.7	too little time is left for more focused preparation for tests and for the final exam.	-2 –1 0 1 2

Your comments: ____

____

____

____

I. On Gender

25. In the KappAbel semifinal the teams will consist of two boys and two girls.

25.1 Do you consider this requirement a good idea? yes: ☐ no: ☐ undecided: ☐

25. 2 Why/ why not?

____

____

____

____

26. In a series of interviews, teachers have been asked about their views on boys' and girls' work with mathematics in class. Please offer comments (in key words) to any statements you find important or provocative.

26.1 **Kjersti:** *It is important that everybody gets to discover that girls are at least as good as boys at mathematics. It is therefore a good idea to have mixed gender groups.*

Key words: ____

____

____

26.2 **Karl:** *Boys and girls have different strengths and weaknesses in mathematics, and should be allowed to cultivate these. It is therefore often an advantage if boys work with boys, and girls work with girls.*

Key words: ____

____

____

26.3 **Elin:** *Gender is irrelevant in mathematics. There is therefore no reason to fuss over whether or not girls and boys are in the same group.*

Key words: ____

____

____

26.4 **Svein:** *Girls may find it difficult to make themselves heard in mathematics. It is therefore a good idea to have boys-only and girls-only groups.*

Key words: ____

____

____

26.5 **Kari:** *It is important that boys and girls learn to mix, treating each other with mutual respect. It is therefore a good idea that boys and girls are in the same group.*

Key words: ____

____

____

27. Think about your grade 9 mathematics students.

Do you perceive differences in terms of whether boys or girls stand out more in the following concrete aspects of the mathematics course? (Please tick.)

		boys	**girls**	**equal**	**undecided**
27.1	Showing interest for mathematics	☐	☐	☐	☐
27.2	Making an effort to get good marks	☐	☐	☐	☐
27.3	Making an effort to get praise	☐	☐	☐	☐
27.4	Co-operation	☐	☐	☐	☐
27.5	Competition	☐	☐	☐	☐
27.6	Deeper understanding	☐	☐	☐	☐
27.7	Practising skills	☐	☐	☐	☐
27.8	Working with numbers	☐	☐	☐	☐
27.9	Working with geometry	☐	☐	☐	☐
27.10	Working with functions	☐	☐	☐	☐
27.11	Working with probability	☐	☐	☐	☐
27.12	Confidence in their own skills in and knowledge of mathematics	☐	☐	☐	☐

Other aspects:

____

___

Optional concluding comments:

___

___

___

___

___

___

___

28. In which county do you work? ___

May we contact you with the puropose of interviewing you and observing how you teach mathematics to a 9th-Year group of students?

If yes, please submit your:

Name: ___

Postal address: ___

___

Email-address: ___

Thank you for your help!

Til matematikklærere som deltar i KappAbel

Det nordiske forskningsprosjektet ”Endring i holdninger og praksisser?” handler om KappAbel. I desember ble det sendt ut et anonymt spørreskjema til alle matematikklærere som underviser i matematikk på 9. trinn, skoleåret 2004/2005.

Vi vil gjerne opprette kontakt med alle matematikklærere som arbeider videre med prosjektarbeidet ”Matematikk og kroppen”. Vi henvender oss derfor til deg og håper at du vil svare på følgende spørsmål:

1. Mener du at deltagelsen i KappAbelkonkurransen, vil ha betydning for matematikkundervisningen i din 9. klasse resten av skoleåret?

Ja: Nei:

Hvis ja, - på hvilken måte (sett kryss):
* Oppgavetyper elevene arbeider med?   Ja: Nei: Vet ikke:

* Måten elevene arbeider på (f.eks. prosjekt)? Ja: Nei: Vet ikke:

* Mengde og type av gruppearbeid?   Ja: Nei: Vet ikke:

* Elevenes holdninger til matematikk?   Ja: Nei: Vet ikke:

2. Har du/dere planlagt å arbeide med prosjektarbeidet ”Matematikk og kroppen”?

Ja: Nei:

3. Hvis svaret er ja, kan vi ta kontakt med deg per e-post?

Ja: Nei:

e-post adresse:

Vi håper at du vil svare direkte (trykk ”svar” /”reply”).

På forhånd takk for hjelpen!

Vennlig hilsen
Tine Wedege
Prosjektleder

PS: Informasjon om forskningsprosjektet finner du på www.matematikksenteret.no under FoU-arbeid. Har du spørsmål, ta gjerne kontakt med meg på tiw@ruc.dk

Appendix 2.3 Email sent Feb-Marts 2005 together with the questionnaire TS2

Kjære matematikklærer!

Takk for din positive tilbakemelding! Vedlagt sender vi et spørreskjema i Word-format, som vi håper du kan fylle ut elektronisk og returnere til Tine.Wedege@matematikksenteret.no .
Hvis du foretrekker å fylle ut et papirskjema, kan du sende en e-mail til Merete.Lysberg@matematikksenteret.no med adressen din, og vi vil sende et skjema til deg i posten.

På forhånd takk!

Vennlig hilsen
Tine Wedege
Prosjektleder

Interviewguide til lærerinterviews v. KappAbel semifinalen
Dato, navne, sted

Tak fordi du ville stille op til det her interview.
Som du ved er vi i gang med et forskningsprojekt om 'Holdninger og praksisser i matematikundervisningen'.
Jeg ville derfor gerne have dig til at fortælle om hvordan du tænker om matematik og matematikundervisning, og hvordan du ser på din egen undervisning i faget.
Hvad du ser som godt i det, du laver? Og om er der andre ting, du er mere utilfreds med?

Jeg skal også sige, at alt hvad du fortæller er anonymt i den forstand, at det ikke komme til at fremstå med navn, skole, el.lign., hvad du har sagt. Det kommer dog til at fremgå af rapporten fra projektet, at interviewet er gennemført ved KappAbel semifinalen, og du vil blive beskrevet med køn og undervisningserfaring.

Er der noget, du gerne vil spørge mig om, før vi går i gang …?

Om et forløb
1.1: Jeg vil altså gerne have dig til at tænke på og fortælle om din undervisning i matematik. For at komme i gang vil jeg bede dig fortælle om et det sidste emne eller forløb, du har arbejdet med i matematik i din 9. klasse. Et forløb som ikke har noget at gøre med KappAbel. Jeg vil gerne få en fornemmelse af, hvad der skete. Hvis jeg nu havde været en flue på væggen, hvad havde jeg så set?

> (Hvis interviewpersonen siger, at nu har de jo lige forberedt sig en masse på KappAbel, så kan man bede dem om at fortælle om forløbet inden).
> Prompts:
> Hvad var det faglige indhold eller det emne, der blev arbejdet med?
> Var det et kapitel i den lærebog, du bruger – hvad er det for resten for en?

Om en normal time i forløbet
1.2: Kan du beskrive en normal time fra det forløb for mig – hvordan foregik det?

> Prompts:
> Hvordan begyndte timerne?
> Hvad arbejdede eleverne med? (hvis svaret inkluderer, at de regner opgaver: var det fra bogen?
> Undervisning kan jo organiseres på mange måder (klassevis, smågrupper, enkeltvis, …), som har forskellige styrker – hvad ville jeg som flue have set i din klasse i netop det forløb?
>
> Probes: (eksempler)
> Du sagde, at du ofte introducerede det nye stof på tavlen – kan du give et eksempel, hvad du mere præcist gjorde?
> Du sagde, at du havde brugt nogle andre opgaver end bogens til at supplere med – kan du huske nogle eksempler på sådanne opgaver? Hvad ville du opnå med de

supplerende opgaver? Du sagde, at du af og til arbejdede med diskussion på hele klassen. Hvad kunne være udgangspunktet for sådan en diskussion? (Kan du give et eksempel på et spørgsmål, du kunne stille som udgangspunkt?).

Uddybning
1.3: Nu er jeg ved at danne mig et billede af, hvordan det foregik [lav en kort beskrivelse med brug af respondentens ord og vendinger]. Men kan du sige lidt mere om [indholdet/opgaverne/den måde eleverne arbejdede på/…].

Den gode time
1.4: Det lyder jo spændende alt sammen. Jeg tror at alle der underviser ved, at det en gang i mellem går som man ønsker sig, men at det andre gange ikke går som planlagt. Prøv at tænke tilbage på en konkret time, hvor du synes det gik rigtig godt Hvad er det der kendetegner sådan en time?

Promts/probes: Kan du uddybe, hvad det var ved den time, der gjorde den god?
Hvad er din egen rolle i sådan en time?
Hvad kendetegner elevernes aktivitet?
Har det her noget med det faglige indhold at gøre – du sagde at du underviste i [sandsynlighedsregning], er det et område du er særligt glad for, eller som særligt inviterer til … ? Er der nogle særlige opgaver eller måder at arbejde på i det her område
(lyt også efter/spørg til arbejdsformer; relationer til anvendelser, herunder andre fag).
Når du siger, at [eleverne var aktive/lærte en masse/ …], hvordan gav det sig så udtryk?
Kan du give et eksempel på det?
Jeg er ved at danne mig et billede af, den her time, kan du fortælle mig lidt mere om, hvad du ser som kvaliteterne i den?

Din typiske måde at undervise på
2. Nu har jeg vist et billede af det forløb, du beskrev for mig før. På hvilke måder mener du, at det du har beskrevet for mig er typisk for den måde du underviser?

Prompts: Hvordan adskiller undervisningen i det her indhold sig evt. fra undervisningen i f.eks. … eller fra andre emner, du underviser i i matematik?
Er undervisningen oftest organiseret på den måde, du har beskrevet med …?
Du nævnte, at du havde brugt nogle supplerende materialer - hvilke forskelle mener du evt. der er mellem behovet for at bruge supplerende materialer i forbindelse med forskellige faglige emner?

Udvikling af undervisningspraksis
3. Det lyder som om du gradvist har udviklet en måde en at undervise på, der er kendetegnet ved … Hvordan har du udviklet den her måde?

Er der noget, der i særlig grad har inspireret dig til at tænke sådan om undervisning i matematik?

Erfaring med KappAbel i år

4. Du har jo i år været involveret i KappAbel projektet, og din 9. klasse har været engageret i KappAbel aktiviteter på forskellige tidspunkter i løbet af året (evt. kort reminder).
Hvad er dine erfaringer med at være med i KappAbel i år?

> Prompts: På hvilke måder oplever du det som foreneligt med det, du gerne vil prioritere i matematikundervisningen?
> Hvordan griber KappAbel aktiviteter evt. forstyrrende ind i det daglige arbejde?
> På hvilke måder er KappAbel evt. også en behagelig afveksling?
> På hvilke måder er KappAbel evt. også en inspirationskilde for din øvrige undervisning?
>
> Lyt efter/spørg til:
> Tværfaglig projekter: er de en berigelse og evt. hvordan? Hvad er styrkerne og svaghederne ved dem?
> matematik som undersøgende proces: Eleverne bliver jo bedt om at undersøge matematik i relation til et emne (i år kroppen, sidste år musik). På hvilke måder adskiller det sig fra de måder, de normalt arbejder på?
> Elevsamarbejde – Er der fordele eller ulemper ved det, som vi ikke allerede har snakket om?
> Kønsspørgsmålet: drenge og piger skal arbejde sammen i finalen – hvordan ser du på det?
> Har drenge og piger typisk forskellige måder at gå ind i KappAbel?

Organisering af KappAbel i år

5. Vil du til sidst fortælle hvordan I har organiseret arbejdet med KappAbel i din 9. klasse og om din egen rolle i forløbet?

Prompts:
Hvordan løste klassen opgaverne i runde 1 og 2?
Hvordan besluttede klassen hvad de ville arbejde med i projektarbejdet? (drenge/piger aktive i beslutningsprocessen?)
Har du givet eleverne feedback f.eks. ved at sige – Det der har ikke noget med matematik at gøre. – Eller: I kan få noget mere matematik ind ved at gøre sådan og sådan.
Har hele klassen været involveret i projektarbejdet?
Hvad har din rolle/funktion været undervejs?

Den lidt mere generelle matematiksituation på skolen

6. Arbejdet med KappAbel har jo foregået i din 9. klasse. Nu er det jo ikke givet, at en idé, der kan bruges på ét klassetrin er god på andre. Jeg vil gerne have dig til at fortælle, om du synes, at det, der lægges vægt på i KappAbel, kan vægtes i andre klasser.

> Promts: Hvis positiv: Er det noget du vil have lyst til at gøre i dine egne andre klasser? Hvordan?

7. På nogle skoler diskuterer lærerne tit matematikundervisning, på andre gør de det aldrig. Hvordan synes du situationen er på din skole, hvad det angår?

> Prompts: Snakker I matematik i frokostpausen? Hvordan beslutter I, hvilke bøger i skal købe? Fortæl mig lidt om sidst I havde et møde, hvad snakkede I om? Er I normalt enige eller uenige, Hvis I diskuterer matematikundervisning?
>
> Hvordan synes du den måde, I tænker om og diskuterer matematikundervisning på på din skole passer med tankerne bag KappAbel?

Afslutning

8. Nu har jeg stillet dig en helt masse spørgsmål. Jeg håber, jeg har givet dig mulighed for at snakke om noget af det, du synes er vigtigt ved matematikundervisningen. Men jeg vil meget gerne have dig til at tilføje sider af matematik i skolen, som jeg ikke har spurgt nok til.

Nasjonalt Senter for Matematikk i Opplæringen

Til matematikklæreren 5. april 2005

Om lærerintervju i forskningsprosjektet *"Endring i holdninger og praksis?"*

Intervjuet inngår i det nordiske forskningsprosjektet "*Endring i holdninger og praksis?*" om matematikkonkurransen KappAbel.

Intervjuet handler om din matematikkundervisning og om erfaringer fra KappAbel. Det varer ca. 30 minutter og gjennomføres av forskningsassistent Kjersti Wæge.

Intervjuet tas opp på disk, og det lages en utskrift. Den sendes deg til orientering og for eventuelle kommentarer, før den bearbeides sammen med ditt utfylte spørreskjema og de øvrige data i forskningsprosjektet.

Ved rapporteringen vil det fremgå at du er en av de 19 matematikklærere fra semifinalen, men din nærmere identitet vil være skjult, bortsett fra kjønn og undervisningsansiennitet.

På forhånd takk for hjelpen!

Med vennlig hilsen
Matematikksenteret

Tine Wedege
Prosjektleder

PS: Informasjon om forskningsprosjektet finner du på www.matematikksenteret.no under FoU-arbeid. Har du spørsmål, ta gjerne kontakt med prosjektleder Tine Wedege på tiw@ruc.dk

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Appendix 3.3

Interviewguide – elevgruppeinterviews

[Interviewet foregår med en gruppe bestående af fire elever fra samme klasse (et semi-finallag).
Kjersti interviewer, mens Tine tager noter (holder bl.a. styr på hvem der tager ordet. Vi skal huske navnesedler.)
Interviewet optages på bånd/diskette, og transkriberes senere.
R = Runde og **F** = Ordet er frit … ved de enkelte spørgsmål]

Velkommen og tak for at I ville være med til det her interview. Vi er i gang med et forskningsprojekt, der handler om matematikundervisning og holdninger til matematik, og vi vil gerne have jeres erfaringer og holdninger med.

Vi skal lige sige, at i rapporten, vi skal skrive, bliver I ikke nævnt med navn eller skole, men der kommer til at stå, at interviewet blev lavet ved KappAbel semifinalen. Her er et informationsbrev som I kan tage med hjem.

Er der nogle af jer, der vil spørge om noet, før vi går i gang?
I er også velkomne til sende os en email med spørgsmål bagefter.

HER GÅR DET EGENTLIGE INTERVIEW I GANG

Det vi gerne vil have jer til at fortælle om er altså jeres erfaringer med matematik og matematikundervisning.

1: Hvad synes I kendetegner **en rigtig god matematiktime**? **R**

Promts: hvor tit sker det, at I har sådan en ’god time’?

2: Kan I beskrive **en ’normal’ matematiktime** for mig? **F**
Hvad gør læreren? Hvad skal I gøre?

Prompts: Hvilken lærebog bruger I normalt?
Hvor meget tid bruger I på at regne opgaver fra bogen?
Bruger læreren også andre materialer eller problemer end dem fra bogen?

3: Da I startede med KappAbel i november, hvordan havde I så evt. forberedt jer på det? **F**
Var det en **forberedelse**, der var meget anderledes end den måde I normalt arbejder på – hvordan ligner den eller ligner den ikke den ’normale’ time, I har fortalt om?

4: Da I gik videre i konkurrencen, brugte I så også tid på at forberede jer specielt på de næste skridt? Adskilte det sig fra de måder I normalt arbejder på? **F**

5: Hvordan har I arbejdet med **KappAbel kravene** – hvem har været involveret i at lave rapporten? Logbogen? Udstilllingen og emballagen? **F**

> Prompts: Hvordan opstod ideen til jeres projekt?
> Hvad har læreren gjort?
> Har han/hun undervist som sædvanlig? Hvordan?

6: Deltagelse i sådan noget som KappAbel kan **opleves** meget forskelligt, f.eks. som **R**

- sjovt,
- irriterende,
- kedeligt,

6.2: Hvordan har I oplevet matematik i KappAbel (opgaver, runde 1, 2 og semifinalen og projektet)? **R**

7: Hvad har det **betydet for jer** at være med? **R**

> Prompts: nu er I jo nået langt, så der er jo ikke så meget at sige til, hvis I synes det har været sjovt på det sidste – har det været det hele vejen.
> Hvordan tror I jeres klassekammerater, der ikke er med her, tænker om det?

8: Hvordan oplever I, at de måder I har arbejdet med KappAbel **adskiller sig fra den måde I normalt arbejder på i matematiktimerne**? **F**
Og hvordan oplever I, at de måder I har arbejdet med KappAbel **ligner** de måder I normalt arbejder på i matematiktimerne?

> Prompts: Har I nogensinde i forløbet tænkt: ”Jamen det her har da ikke noget med matematik at gøre!”

9: Er der noget I har tænkt på før interviewet – eller under interviewet – ”Det her vil jeg gerne sige”, og som I så ikke har fået sagt? **F**

Appendix 3.4

Nasjonalt Senter for Matematikk i Opplæringen

Til semifinalelaget i KappAbel 5. april 2005

Om gruppeintervju i forskningsprosjektet *"Endring i holdninger og praksis?"*

Intervjuet inngår i det nordiske forskningsprosjektet *"Endring i holdninger og praksis?"* om matematikkonkurransen KappAbel.

Intervjuet handler om matematikkundervisningen i skolen og om elevenes erfaringer med Kapp Abel. Det er et gruppeintervju med et semifinalelag, og det gjennomføres av prosjektmedarbeider Kjersti Wæge.

Intervjuet tas opp på disk, og det lages en utskrift, som sendes til dere før den bearbeides. Vi ser gjerne at dere kommenterer utskriften.

I forskningsrapporten vil det fremgå at dere er et semifinalelag i KappAbel, men nærmere identitet vil være skjult, bortsett fra kjønn. Navnene vil bli endret.

På forhånd takk for hjelpen!

Med vennlig hilsen

Tine Wedege
Prosjektleder

PS: Informasjon om forskningsprosjektet finner du på www.matematikksenteret.no under FoU-arbeid. Har du spørsmål, ta gjerne kontakt med prosjektleder Tine Wedege på tiw@ruc.dk

Nasjonalt Senter for Matematikk i Opplæringen

Til elevens foresatte

9. mai 2005

Om gruppeintervju i forskningsprosjektet ”*Endring i holdninger og praksis?*”

Elevene på semifinalelaget i KappAbel ble intervjuet under finaledagene i Arendal. Intervjuet inngår i det nordiske forskningsprosjektet ”*Endring i holdninger og praksis?*” om matematikkonkurransen KappAbel. Prosjektet er initiert av Nordisk Kontakt Komité for ICME-10 og finansiert av Nordisk Ministerråd og Nasjonalt Senter for Matematikk i Opplæringen.

Prosjektet omfatter en landsdekkende spørreskjemaundersøkelse blant matematikklærere på 9. trinn (desember 2004), intervju med elever og lærere som deltar i KappAbel og observasjoner i matematikktimer (2005).

Gruppeintervjuet handlet om matematikkundervisningen i skolen og om elevenes erfaringer med KappAbel, og det ble gjennomført av forskningsmedarbeider Kjersti Wæge.

Intervjuet ble tatt opp på bånd, og det vil bli laget en utskrift, som sendes til elevene for kommentarer, før den bearbeides.

I forskningsrapporten er elevene anonyme. Det vil fremgå at de er et av semifinalelagene i KappAbel, men nærmere identitet vil være skjult, bortsett fra kjønn. Navnene vil derfor bli endret.

Vi håper at dere vil godkjenne at vi bruker gruppeintervjuet i forskningsprosjektet. Hvis ikke, ber vi dere gi beskjed til Kjersti Wæge (tlf.: 73 55 11 45 eller e-post: kjersti.wege@matematikksenteret.no).

Med vennlig hilsen

Tine Wedege
Prosjektleder

PS: Informasjon om forskningsprosjektet finner du på www.matematikksenteret.no under FoU-arbeid. Har dere spørsmål, ta gjerne kontakt med prosjektleder Tine Wedege på tiw@ruc.dk

Appendix 3.6

Nasjonalt Senter for
Matematikk i Opplæringen

Til læreren

10. oktober 2005

Intervju i forskningsprosjektet *"Endring i holdninger og praksis?"*

Vi har nå laget en utskrift av intervjuet Kjersti gjorde med deg under finaledagene i Arendal, og vi ber deg om å lese gjennom denne utskriften. Hvis du har noen kommentarer eller bemerkninger til intervjuet, ber vi deg om å skrive kommentarene på baksiden av dette arket eller i selve intervjuet. Legg deretter arkene i den vedlagte konvolutten og send den til oss. Porto er betalt.

Vedlagt er en konvolutt med et tilsvarende infoskriv til elevene som deltok på gruppeintervjuet. Vi ber deg om å gi dem konvolutten og sende deres kommentarer sammen med dine kommentarer.

Hvis du har spørsmål, kan du ta kontakt med Kjersti på tlf.: 73 55 11 45 eller e-post: kjersti.wege@matematikksenteret.no

På forhånd tusen takk!

Med vennlig hilsen

Tine Wedege
Prosjektleder

PS: Informasjon om forskningsprosjektet finner du på www.matematikksenteret.no under FoU-arbeid. Har du spørsmål, ta gjerne kontakt med prosjektleder Tine Wedege på tiw@ruc.dk

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